EXPONENTS AND RADICALS

Exponents

Exponents tell how many times to use a number itself in multiplication. There are different laws that guides in calculations involving exponents. In this chapter we are going to see how these laws are used.

For example; $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$ and $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$, we have multiplied alike factors and got answers which are 4^6 and 5^7. 4 and 5 are our factors and they are called bases while 6 and 7 are called exponents. 4^6 is read as 'sixth power of four' or 'four to the sixth power' and $5^7$is read as 'seventh power of five' or 'five to the seventh power'. $4 \times 4 \times 4 \times 4 \times 4 \times 4$ is the expanded form of 4^6 and the expanded form of 5^7 is $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$. The product of any expanded form is called a power of the factor. 4^6 and 5^7 are the powers of our factors. 4^6 means multiply 4 six times and 5^7 means multiply 5 seven times.

Indication of power, base and exponent is done as follows:

a. 6^2
b. 9^3
c. 2^8
d. 4^9
e. 5^3
f. 10^5

Solution:

a. 6^2 is a power, 6 is a base and 2 is the exponent.
b. 9^3 is a power, 9 is a base and 3 is the exponent.
c. 2^8 is a power, 2 is a base and 8 is the exponent.
d. 4^9 is a power, 4 is a base and 9 is the exponent.
e. 5^3 is a power, 5 is a base and 3 is the exponent.
f. 10^5 is a power, 10 is a base and 5 is the exponent.

To write the expanded form of the following powers:

a. 7^3
b. 205^2
c. 16^4
d. $(-4)^6$
e. $(\frac{1}{2})^8$
f. m^5

Solution

a. $7^3 = 7 \times 7 \times 7$

b. $205^2 = 205 \times 205$

c. $16^4 = 16 \times 16 \times 16 \times 16$

d. $(-4)^6 = -4 \times -4 \times -4 \times -4 \times -4 \times -4$

e. $(\frac{1}{2})^8 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

f. $m^5 = m \times m \times m \times m \times m$

To write each of the following in power form:

a. $10 \times 10 \times 10 \times 10 \times 10 \times 10$

b. $\frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7}$

c. $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

d. $-8 \times -8 \times -8$

Soln.

a. $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

b. $\frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} = (\frac{-3}{7})^4$

c. $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

d. $-8 \times -8 \times -8 = (-8)^3$

The Laws of Exponents

List the laws of exponents

First law: Multiplication of positive integral exponent

$$x^m \times x^n = x^{m+n}$$

Second law: Division of positive integral exponent

$$\frac{x^m}{x^n} = x^{m-n}$$

Third law: Zero exponents

$$x^0 = 1.$$

Fourth law: Negative integral exponents

$\frac{1}{x^1} = x^{-1}$.

Verification of the Laws of Exponents

Verify the laws of exponents

First law: Multiplication of positive integral exponent

For example; if you are given $4^3 \times 4^6$, in expanded form you may write it as

$$\underbrace{4 \times 4 \times 4}_{\text{3 factors}} \times \underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{\text{6 factors}}.$$

There are 9 factors in total.

Therefore $4^3 \times 4^6 = 4^9$.

Generally, when we multiply powers having the same base, we add their exponents. If x is any base and m and n are the exponents, therefore:

$$x^m \times x^n = x^{m+n}$$

Example 1

a. $2^4 \times 2^6$

b. $(0.25)^3 \times (0.25)^9$

c. $(\frac{-7}{11})^7 \times (\frac{-7}{11})^{12}$

d. $a^6 \times a^8 \times a^2$

Solution

a. $2^4 \times 2^6 = 2^{4+6} = 2^{10}$

b. $(0.25)^3 \times (0.25)^9 = (0.25)^{3+9} = (0.25)^{3+9}$

c. $(\frac{-7}{11})^7 \times (\frac{-7}{11})^{12} = (\frac{-7}{11})^{7+12} = (\frac{-7}{11})^7$

d. $a^6 \times a^8 \times a^2 = a^{6+8+2} = a^{16}$

If you are to write the expression using the single exponent, for example, $(6^3)^4$. The expression can be written in expanded form as:

$$(6^3)^4 = \underbrace{6^3 \times 6^3 \times 6^3 \times 6^3}_{\text{4 factors}}$$

$$= \underbrace{\underbrace{6 \times 6 \times 6}_{\text{3 factors}} \times \underbrace{6 \times 6 \times 6}_{\text{3 factors}} \times \underbrace{6 \times 6 \times 6}_{\text{3 factors}} \times \underbrace{6 \times 6 \times 6}_{\text{3 factors}}}_{\text{4 factors of 3 factors = 12 factors}}$$

$= 6^{12}$ Therefore, $6^{12} = 6^{3 \times 4} = 6^{12}$

Generally if a and b are real numbers and n is any integer,

$$(a \times b)^n = a^n \times b^n$$

Example 2

$$(y^4)^8 = (y)^{4 \times 8} = (y)^{32}$$

Example 3

$$((\tfrac{3}{4})^3)^6 = (\tfrac{3}{4})^{3 \times 6} = (\tfrac{3}{4})^{18}$$

Example 4

$$((-0.75)^7)^4 = (-0.75)^{7 \times 4} = (-0.75)^{28}$$

Generally, $(x^m)^n = X^{(mxn)}$

Example 5

Rewrite the following expressions under a single exponent for those with identical exponents:

a. $z^4 \times 5zx^2$
b. x^2y^2

Solution

a. $z^4 \times 5zx^2 = z \times z \times z \times z \times 5 \times z \times x \times x$

$= 5 \times z \times z \times z \times z \times z \times x \times x$

$= 5 \times z^5 \times x^2 = 5z^5x^2$

b. $x^2y^2 = (xy)^2$

Second law: Division of positive integral exponent

Example 6

The expression $\frac{5^6}{5^2}$ can be written as:

$$\frac{5^6}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5}$$

$$= 5 \times 5 \times 5 \times 5$$

$$= 5^4$$

Which is the same as:

$$\frac{5^6}{5^2} = 5^{6-2} = 5^4$$

Example 7

$\frac{k^5}{k^4}$

Solution

$$\frac{k^5}{k^4} = \frac{k \times k \times k \times k \times k}{k \times k \times k \times k}$$

$$= k$$

Which is the same as:

$$\frac{k^5}{k^4} = k^{5-4}$$

$$= k$$

Therefore, to divide powers of the same base we subtract their exponents (subtract the exponent of the divisor from the exponent of the dividend). That is,

$$\frac{x^m}{x^n} = x^{m-n}$$

where x is a real number and $x \neq 0$, m and n are integers. m is the exponent of the dividend and n is the exponent of the divisor.

Example 8

a. $\frac{10^8}{10^5}$

b. $\frac{5a^6}{a^4}$

c. $\frac{81}{3^2}$

d. $\frac{((3)^2)^5}{3^4}$

e. $18b^4 \div (4b^3 - 2b^3)$

solution

a. $\frac{10^8}{10^5} = 10^{8-5} = 10^3$

b. $\frac{5a^6}{a^4} = 5a^{6-4} = 5a^2$

c. $\frac{81}{3^2} = \frac{3^4}{3^2} = 3^{4-2} = 3^2$

d. $\frac{(3^2)^5}{3^4} = \frac{3^{2\times 5}}{3^4}$

$= \frac{3^{10}}{3^4} = 3^{10-4}$

$= 3^6$

e. $18b^4 \div (4b^3 - 2b^3)$

$= 18b^4 \div 2b^3$

$= \frac{18b^4}{2b^3}$

$= 9b^{4-3}$

$= 9b$

Third law: Zero exponents

Example 9

$$\frac{2^5}{2^5} = \frac{2\times 2\times 2\times 2\times 2}{2\times 2\times 2\times 2\times 2} = 1$$

This is the same as:

$$\frac{2^5}{2^5} = 2^{5-5} = 2^0 = 1$$

If a ≠ 0, then

$$\frac{a^3}{a^3} = a^{3-3} = a^0 = 1.$$

Which is the same as:

$$\frac{a^3}{a^3} = \frac{a\times a\times a}{a\times a\times a} = 1$$

Therefore if *x* is any real number not equal to zero, then $X^0 = 1$,Note that $0^0$is undefined (not defined).

Fourth law: Negative integral exponents

For example, if we simplify the expression $\frac{9^3}{9^7}$ we obtain

$\frac{9^3}{9^7} = \frac{9 \times 9 \times 9}{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9} = \frac{1}{9^4}$ .... (i)

It is the same as:

$\frac{9^3}{9^3} = 9^{3-7} = 9^{-4}$ .... (ii)

Equating (i) and (ii) since the expressions are identical, then

$\frac{1}{9^4} = 9^{-4}$

Another example, $\frac{d^4}{d^6}$, $d \neq 0$. We can simplify the expression as follows:

$\frac{d^4}{d^6} = \frac{d \times d \times d \times d}{d \times d \times d \times d \times d \times d} = \frac{1}{d^2}$ .... (i)

Also;

$\frac{d^4}{d^6} = d^{4-6} = d^{-2}$ .... (ii)

Equating (i) and (ii) because they are identical expressions, we obtain

$\frac{1}{d^2} = d^{-2}$

Generally, $\frac{1}{x^n} = x^{-n}$ if $x \neq 0$ and n is an integer. If $n = 1$, then $\frac{1}{x^1} = x^{-1}$. But $\frac{1}{x^1} = \frac{1}{x}$ and $\frac{1}{x}$ is called a reciprocal of x and is written as x^{-1}.

Example 10

Express the following powers using positive exponents:

a. a^{-7}

b. 2^{-3}

c. $\frac{1}{4^{-2}}$

c. 5^{-8}

d. $(\frac{1}{2})^{-4}$

Solution

a. $a^{-7} = \frac{1}{a^7}$

b. $2^{-3} = \frac{1}{2^3}$

d. $5^{-8} = \frac{1}{5^8}$

e. $(\frac{1}{2})^{-4} = \frac{1}{2^{-4}} = \frac{1}{\frac{1}{2^4}}$

$= 1 \div \frac{1}{2^4}$

$= 1 \times \frac{2^4}{1}$

$= 2^4$

c. $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 1 \div \frac{1}{4^2} = 1 \times \frac{4^2}{1} = 4^2$

Exercise 1

1. Indicate base and exponent in each of the following expressions:

a. 4^{670}

b. m^{x+y}

c. $(w+z)^n$

d. 990^{-72}

2. Write each of the following expressions in expanded form:

a. $(-7)^4$

b. $(\frac{1}{8})^6$

c. 12^{-3}

d. $(0.72)^{-4}$

3. Write in power form each of the following numbers by choosing the smallest base:

a. 169

b. 81

c. 10,000

d. 625

a. 169 b. 81c. 10 000 d. 625

4. Write each of the following expressions using a single exponent:

a. $(12)^{15} \times (12)^{25}$

b. $10^5 \times 10^n \times 10^{-m}$

c. $y^6 \times y^8$

d. $(\frac{1}{6})^3 \times (\frac{1}{6}) \times (\frac{1}{6})^6$

5. Simplify the following expressions:

a. $28a^4b^6 \div 7a^{-3}b^{-4}$

b. $\frac{625r^4}{25r^2}$

c. $\frac{x^7y^9}{x^{-5}y^{-1}}$

d. $\frac{(3s^2)^6}{(3s)^4}$

6. Solve the following equations:

a. $3^m = 3^{2+5}$

b. $7^{n-2} = 7^8$

c. $(\frac{1}{9})^{-x} = 81$

d. $7056 = (2x)^2$

7. Express 64 as a power with:

1. Base 4
2. Base 8
3. Base 2

Base 4 Base 8 Base 2

8. Simplify the following expressions and give your answers in either zero or negative integral exponents.

a. $\frac{\pi r^2 h}{\pi r^5 r^2 h^5}$

b. $\frac{4b^2 \times b^5}{b^4 \times b}$

9. Give the product in each of the following:

a. 6^3

b. $(\frac{-1}{2})^4$

c. $5 \times 5 \times 5 \times 5 \times 5$

d. 2^5

10. Write the reciprocal of the following numbers:

a. 19

b. d where d is real

c. $\frac{1}{8}$

d. $(\frac{1}{9})^{-2}$

Laws of Exponents in Computations

Apply laws of exponents in computations

Example 11

a. $\frac{10^8}{10^5}$

b. $\frac{5a^6}{a^4}$

c. $\frac{81}{3^2}$

d. $\frac{((3)^2)^5}{3^4}$

e. $18b^4 \div (4b^3 - 2b^3)$

Solution

a. $\frac{10^8}{10^5} = 10^{8-5} = 10^3$

b. $\frac{5a^6}{a^4} = 5a^{6-4} = 5a^2$

c. $\frac{81}{3^2} = \frac{3^4}{3^2} = 3^{4-2} = 3^2$

d. $\frac{(3^2)^5}{3^4} = \frac{3^{2\times 5}}{3^4}$
$= \frac{3^{10}}{3^4} = 3^{10-4}$
$= 3^6$

e. $18b^4 \div (4b^3 - 2b^3)$
$= 18b^4 \div 2b^3$
$= \frac{18b^4}{2b^3}$
$= 9b^{4-3}$
$= 9b$

Radicals

Radicals

Simplify radicals

Radicals are opposite of exponents. For example when we raise 2 by 2 we get 4 but taking square root of 4 we get 2. The same way we can raise the number using any number is the same way we can have the root of that number. For example, square root, Cube root, fourth root, fifth roots and so on. We can simplify radicals if the number has factor with root, but if the number has factors with no root then it is in its simplest form. In this chapter we are going to learn how to find the roots of the numbers and how to simplify radicals.

When a number is expressed as a product of equal factors, each of the factors is called the root of that number. For example,25 = 5× 5;so, 5 is a square root of 25: 64 = 8× 8; 8 is a square root of 64: 216 = 6 ×6 ×6, 6 is a cube root of 216: 81 = 3 × 3 ×3 ×3,3 is a fourth root of 81: 1024 = 4 ×4 ×4 ×4 ×4, 4 is a fifth root of 1024.

Therefore, the nth root of a number is one of the n equal factors of that number. The symbol for nth root is $\sqrt[n]{}$ where√is called a radical and n is the index (indicates the root you have to find). If the index is 2, the symbol represents square root of a number and it is simply written as√without the index 2. $\sqrt[n]{p}$ is expressed in power form as,

$(p)^{\frac{1}{n}}$. For example $\sqrt{4} = 4^{\frac{1}{2}}$: $\sqrt[4]{64} = 64^{\frac{1}{4}}$.

nth root of a number by prime factorization

Example 1, simplify the following radicals

a. $\sqrt{144}$
b. $\sqrt{225}$
c. $\sqrt[3]{1000}$
c. $\sqrt[5]{32}$
d. $\sqrt[3]{72}$

Soution

Write each number as a product of its prime factors

a. $\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$

because we are required to find square root, group the factors into groups of 2 equal factors.

$= \sqrt{(2 \times 2) \times (2 \times 2) \times (3 \times 3)}$

for each group take only one of the factors

$= 2 \times 2 \times 3$

$= 12$

b. $\sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5}$

group the factors into groups of two equal factors since we are required to find the square root of a number.

$= \sqrt{(3 \times 3) \times (5 \times 5)}$

from each group, take one factor

$= 3 \times 5$

$= 15$

c. $\sqrt[3]{1000} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

group the factors into groups of 3 equal factors.

$= \sqrt[3]{(2 \times 2 \times 2) \times (5 \times 5 \times 5)}$

take one factor from each group

$= 2 \times 5$

$= 10$

d. $\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$

group the factors into groups of 5 equal factors

$= \sqrt[5]{(2 \times 2 \times 2 \times 2 \times 2)}$

take one factor from each group

$= 2$

e. $\sqrt[3]{72} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3}$

group the factors into groups of 3 equal factors

$= \sqrt[3]{(2 \times 2 \times 2) \times 3 \times 3}$

$= 2\sqrt[3]{3 \times 3}$

$= 2\sqrt[3]{9}$

Example 2

a. $\sqrt{7} \times \sqrt{3}$
b. $3\sqrt[3]{2}$

Solution

a. $\sqrt{7} \times \sqrt{3} = \sqrt{7 \times 3}$
$= \sqrt{21}$

b. $3\sqrt[3]{2} = \sqrt[3]{3 \times 3 \times 3 \times 2}$
$= \sqrt[3]{54}$

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} = \sqrt{ab}$ and $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

Basic Operations on Radicals

Perform basic operations on radicals

Different operations like addition, multiplication and division can be done on alike radicals as is done with algebraic terms.

Simplify the following:

a. $3\sqrt{5} + 7\sqrt{5}$
b. $4\sqrt{18} - 3\sqrt{8}$
c. $16 + \sqrt{121} - 2\sqrt[3]{2197}$
d. $2\sqrt{3} - \sqrt[3]{8} - 3.$

Solution

a. $3\sqrt{5} + 7\sqrt{5} = (3 + 7)\sqrt{5}$
$= 10\sqrt{5}$

b. $4\sqrt{18} - 3\sqrt{8} = 4\sqrt{2 \times 3 \times 3} - 3\sqrt{2 \times 2 \times 2}$
$= 3 \times 4\sqrt{2} - 2 \times 3\sqrt{2}$
$= 12\sqrt{2} - 6\sqrt{2}$
$= 6\sqrt{2}$

c. $16 + \sqrt{121} - 2\sqrt[3]{2197} = 16 + \sqrt{11 \times 11} - 2\sqrt[3]{13 \times 13 \times 13}$
$= 16 + 11 - (2 \times 13)$
$= 27 - 26$
$= 1$

Simplify each of the following expressions:

a. $\sqrt{3}(\sqrt{15} - \sqrt{13}) + \sqrt{5}$

b. $\sqrt{27} \times \sqrt{48}$

c. $\frac{\sqrt[3]{64}}{2\sqrt{25}}$

d. $\sqrt{\frac{49}{169}}$

e. $\frac{2\sqrt{3} \times 2\sqrt{5}}{\sqrt{12} \times 3\sqrt{15}}$

Solution

a. $\sqrt{3}(\sqrt{15} - \sqrt{13}) + \sqrt{5} = \sqrt{3} \times \sqrt{15} - \sqrt{3} \times \sqrt{13} + \sqrt{5}$
$= \sqrt{3 \times 15} - \sqrt{3 \times 13} + \sqrt{5}$
$= \sqrt{3 \times 3 \times 5} - \sqrt{39} + \sqrt{5}$
$= 3\sqrt{5} - \sqrt{39} + \sqrt{5}$
$= 4\sqrt{5} - \sqrt{39}$

b. $\sqrt{27} \times \sqrt{48} = \sqrt{27 \times 48}$
$= \sqrt{3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3}$

group the factors into groups of two equal factors and from each group take one of the factors.

$$= 3 \times 3 \times 2 \times 2$$
$$= 36$$

c. $$\frac{\sqrt[3]{64}}{2\sqrt{25}} = \frac{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}}{2\sqrt{5 \times 5}}$$
$$= \frac{2 \times 2}{2 \times 5}$$
$$= \frac{2}{5}$$

d. $$\sqrt{\frac{49}{169}} = \sqrt{\frac{7 \times 7}{13 \times 13}}$$
$$= \frac{7}{13}$$

e. $$\frac{2\sqrt{3} \times 2\sqrt{5}}{\sqrt{12} \times 3\sqrt{15}} = \frac{2\sqrt{3} \times 2\sqrt{5}}{\sqrt{2 \times 2 \times 3} \times 3\sqrt{15}}$$
$$= \frac{2\sqrt{3} \times 2\sqrt{5}}{2\sqrt{3} \times 3\sqrt{15}}$$
$$= \frac{2\sqrt{5}}{3\sqrt{15}}$$

The Denominator

Rationalize the denominator

If you are given a fraction expression with radical value in the denominator and then you express the expression given in such a way that there are no radical values in the denominator, the process is called rationalization of the denominator.

Example 12

Rationalize the denominator of the following expressions:

c. $\frac{4}{\sqrt{24}} = \frac{4}{\sqrt{24}} \times \frac{\sqrt{24}}{\sqrt{24}}$ (multiply by $\sqrt{24}$ up and down)

$$= \frac{4\sqrt{2 \times 2 \times 2 \times 3}}{\sqrt{24 \times 24}}$$

$$= \frac{4 \times 2\sqrt{6}}{24}$$

$$= \frac{\sqrt{6}}{3}$$

Generally, if we rationalize $\frac{1}{\sqrt{a}}$, we simply multiply by $\frac{\sqrt{a}}{\sqrt{a}}$. That is:

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$$

Example 13

Rationalize the denominator for each of the following expressions:

a. $\frac{2}{\sqrt{3} + \sqrt{4}}$

b. $\frac{2 - \sqrt{5}}{2\sqrt{6} - \sqrt{7}}$

c. $\frac{3\sqrt{7}}{\sqrt{7} + \sqrt{15}}$

d. $\frac{\sqrt{2} + 4\sqrt{5}}{\sqrt{3} - \sqrt{15}}$

Solution

To rationalize these fractions, we have to multiply by the fraction that is equals to 1. The factor should be considered by referring the difference of two squares.

a. $\frac{2}{\sqrt{3}+\sqrt{6}}$ (our fraction have denominator $\sqrt{3}+\sqrt{4}$, from the difference of two squares, we will take $\sqrt{3}$ - $\sqrt{4}$, therefore our rationalizing factor is $\frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$)

thus,

$$\frac{2}{\sqrt{3}+\sqrt{6}} = \frac{2}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$$

$$= \frac{2(\sqrt{3}-6)}{(\sqrt{3}+\sqrt{6})(\sqrt{3}-\sqrt{6})}$$

$$= \frac{2\sqrt{3}-2\sqrt{6}}{(\sqrt{3})^2-\sqrt{3}\times\sqrt{6}+\sqrt{6}\times\sqrt{3}-(\sqrt{6})^2}$$

$$= \frac{2\sqrt{3}-2\sqrt{6}}{3-6}$$

$$= \frac{2\sqrt{3}-2\sqrt{6}}{-3}$$

$$= \frac{2(\sqrt{6}-\sqrt{3})}{3}$$

b. $\frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}}$ rationalizing factor is $2\sqrt{6}+\sqrt{7}$

thus,

$$\frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}} = \frac{2-\sqrt{5}}{2\sqrt{6}-\sqrt{7}} \times \frac{2\sqrt{6}+\sqrt{7}}{2\sqrt{6}+\sqrt{7}}$$

$$= \frac{2(2\sqrt{6}+\sqrt{7})-\sqrt{5}(2\sqrt{6}+\sqrt{7})}{(2\sqrt{6}-\sqrt{7})(2\sqrt{6}+\sqrt{7})}$$

$$= \frac{4\sqrt{6}+2\sqrt{7}-2\sqrt{5\times6}-\sqrt{5\times7}}{(2\sqrt{6})^2-(\sqrt{7})^2}$$

$$= \frac{4\sqrt{6}+2\sqrt{7}-2\sqrt{30}-\sqrt{35}}{4\times6-7}$$

$$= \frac{4\sqrt{6}+2\sqrt{7}-2\sqrt{30}-\sqrt{35}}{17}$$

c. $\frac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}}$, rationalizing factor is $\sqrt{7}$ - $\sqrt{15}$

thus,

$$\frac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}} = \frac{3\sqrt{7}}{\sqrt{7}+\sqrt{15}} \times \frac{\sqrt{7}-\sqrt{15}}{\sqrt{7}-\sqrt{15}}$$

$$= \frac{3\sqrt{7\times7}-3\sqrt{7\times15}}{(\sqrt{7})^2-(\sqrt{15})^2}$$

$$= \frac{3\times7-3\times\sqrt{105}}{7-15}$$

$$= \frac{21-3\sqrt{105}}{-8}$$

$$= \frac{3\sqrt{105}-21}{8}$$

d. $\frac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-\sqrt{15}}$, rationalizing factor is $\sqrt{3}$ + $\sqrt{15}$

thus,

$$\frac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-\sqrt{15}} = \frac{\sqrt{2}+4\sqrt{5}}{\sqrt{3}-15} \times \frac{\sqrt{3}+\sqrt{15}}{\sqrt{3}+\sqrt{15}}$$

$$= \frac{\sqrt{2}\left(\sqrt{3}+\sqrt{15}\right)+4\sqrt{5}(\sqrt{3}+\sqrt{15})}{\left(\sqrt{3}-\sqrt{15}\right)(\sqrt{3}+\sqrt{15})}$$

$$= \frac{\sqrt{2\times3}+\sqrt{2\times15}+4\sqrt{5\times3}+4\sqrt{5\times15}}{(\sqrt{3})^2-(\sqrt{15})^2}$$

$$= \frac{\sqrt{6}+\sqrt{30}+4\sqrt{15}+4\times5\sqrt{3}}{3-15}$$

$$= \frac{\sqrt{6}+\sqrt{30}+4\sqrt{15}+20\sqrt{3}}{-12}$$

Exercise 2

1. Simplify each of the following by making the number inside the radical sign as small as possible:

a. $\sqrt{270}$
b. $\sqrt{98}$
c. $\sqrt{1008}$

2. Solve the equation: $\sqrt[3]{12} = 12^x$

3. Express each of the following under a single radical sign:

a. $\sqrt{12} \times 4\sqrt{38}$
b. $\sqrt{50} \times \sqrt{98}$

4. Write the following expressions without a radical sign:

a. $\sqrt[4]{16m^4}$
b. $\sqrt{32} \times \sqrt{8}$

5. Perform the following operations:

a. $2\sqrt{5}(\sqrt{40} \times \sqrt{25})$
b. $(3\sqrt{12})^2$
c. $3(12 - 7\sqrt{5})$
d. $6\sqrt{3} + \sqrt{3} - 2\sqrt{3}$
e. $\sqrt{a-b} + \sqrt{36a^2 - 0} - b$
f. $\frac{\sqrt[3]{64}}{\sqrt{81}}$

6. Rationalize the denominator in each of the following:

a. $\frac{2}{2\sqrt{a}+\sqrt{b}}$
b. $\frac{6+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$
c. $\frac{\sqrt{6}}{\sqrt{3}+2}$
d. $\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$
e. $\frac{2\sqrt{7}-\sqrt{5}}{2\sqrt{5}-\sqrt{3}}$
f. $\frac{\sqrt{8}}{\sqrt{9}}$

Square Roots and Cube Roots of Numbers from Mathematical Tables

Read square roots and cube roots of numbers from mathematical tables

If you are to find a square root of a number by using Mathematical table, first estimate the square root by grouping method. We group a given number into groups of two numbers from right. For example; to find a square root of 196 from the table, first we have to group the digits in twos from right i.e. 1 96. Then estimate the square root of the number in a group on the extreme left.

In our example it is 1. The square root of 1 is 1. Because we have two groups, this means that the answer has two digits before the decimal point. Our number is 196, read 1.9 in the table on the extreme left. From our number, we are remaining with 6, now look at the column labeled 6. Read the number where the row of 1.9 meet the column labeled 6. It meets at 1.400. Therefore the square root of 196 = 14.00 since we said that the answer must have 2 digits before decimal points. Note: If you are given a number with digits more than 4 digits. First, round off the number to four significant figures and then group the digits in twos from right. For example; the number 75678 has five digits. When we round it off into 4 digits we get 75680 and then grouping into two digits we get 7 56 80. This shows that our answer has 3 digits. We start by estimating the square root of the number to the extreme left, which is 7, the square root of 7 is between 2 and 3. Using the table, along the row 7.5, look at the column labeled 6. Read the answer where the row 7.5 meets the column 6. Then go to where it is written mean difference and look at the column labeled 8, read the answer where it meets the row 7.5. Take the first answer you got where the row 7.5 met the column 6 and add with the second answer you got where the row 7.5 met the column 8 (mean difference column). The answer you get is the square root of 75680. Which is 275.1

Transposition of Formula

Re-arranging Letters so that One Letter is the Subject of the Formula

Re-arrange letters so that one letter is the subject of the formula

A formula is a rule which is used to calculate one quantity when other quantities are given. Examples of formulas are:

a. $A = \frac{1}{2}bh$

b. $P = 2(w + l)$

c. $V = \pi r^2 h$

d. $I = \frac{PRT}{100}$

A formula can be expressed in different ways. For example; $V = \pi r^2 h$ can be written in the following ways: (i) $h = \frac{V}{\pi r^2}$ (ii) $\pi = \frac{V}{r^2 h}$ (iii) $r = \sqrt{\frac{V}{\pi h}}$. These ways of expressing the formulas in different ways is called transposition of the formula or make one of the symbols a subject of the formula.

Example 14

From the following formulas, make the indicated symbol a subject of the formula:

a. $V = \pi r^2 h$ (h)

b. $I = \frac{PRT}{100}$ (p)

c. $T = 2\pi\sqrt{\frac{l}{g}}$ (g)

d. $S = \frac{1}{2}at^2$ (a)

Solution

a. $V = \pi r^2 h$

divide by πr^2 both sides

$\frac{V}{\pi r^2} = h$

Therefore $h = \frac{V}{\pi r^2}$

b. $\mathbf{I = \frac{PRT}{100}}$

$100I = PRT$ (multiply by 100 both sides)

$\frac{100I}{RT} = P$ (divide by RT both sides)

Therefore, $P = \frac{100I}{RT}$

c. $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$ (divide by 2π both sides)

$\frac{T^2}{4\pi^2} = \frac{l}{g}$ (square both sides)

$T^2 g = 4\pi^2 l$ (cross multiplication)

$g = \frac{4\pi^2 l}{T^2}$ (divide by T^2 both sides)

Therefore, $g = \frac{4\pi^2 l}{T^2}$

Transposing a Formulae with Square Roots and Square

Transpose a formulae with square roots and square

Make the indicated symbol a subject of the formula:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (g)$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}} \text{ (divide by } 2\pi \text{ both sides)}$$

$$\frac{T^2}{4\pi^2} = \frac{l}{g} \text{ (square both sides)}$$

$$T^2g = 4\pi^2 l \text{ (cross multiplication)}$$

$$g = \frac{4\pi^2 l}{T^2} \text{ (divide by } T^2 \text{ both sides)}$$

Therefore, $g = \frac{4\pi^2 l}{T^2}$

Exercise 3

1. Change the following formulas by making the given letter as the subject of the formula.

a. $A = P + \frac{PRT}{100}$ (P)

b. $S = \frac{f}{gh}$ (h)

c. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (u)

d. $P = W(\frac{1+a}{1-a})$ (a)

2. Use mathematical tables to find square root of each of the following:

a. $\sqrt{441}$

b. $\sqrt{2025}$

c. $\sqrt{81.99}$

d. $\sqrt{0.0004}$