THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours **Wednesday, 08th February 2012 a.m.**

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

1. Use a non programmable calculator to evaluate correctly to four decimal places the values of the following:

(a) $\sqrt[4]{\dfrac{(41.67)^8 \times (34.35)^3}{(2.351)^4}}$

(b) $\sqrt[5]{\dfrac{(46.95)^6 \times (\sin 57.56°)^3}{(\cos 68.5°)^2 \times \sqrt{164.8}}}$

(c) $M = \sqrt{\dfrac{s(s-a)}{(s-b)(s-c)}}$ where $a = 6.5877$, $b = 7.8498$, $c = 8.6074$ and $s = 11.5225$

(10 marks)

2. (a) Solve the equation $3\sinh x - \cosh x = 1$.

(b) Show that $\cosh^{-1}(x^2)$, can be expressed as $x^2 + \sqrt{x^4 - 1}$.

(c) Prove the identity $\cosh^2 \beta - \sinh^2 \beta = 1$.

(10 marks)

3. Upendo project, makes two kinds of mixture for planting: gardening mixture and plotting mixture. A package of gardening mixture requires 2 kg of soil, 1 kg of peat moss and 1 kg of fertilizer. A package of plotting mixture requires 1 kg of soil, 2 kg of peat moss and 3 kg of fertilizer. She has at most 16 kg of soil, 11 kg of peat moss and 15 kg of fertilizer. A package of gardening mixture sells for Tshs.5000/= and a package of plotting mixture sells for Tshs.8000/=. How many packages of each type of mixture should be made to maximize revenue?

(10 marks)

4. (a) The following table shows distribution of marks on a final examination in Advanced Calculus.

Marks	Number of students
90 – 99	9
80 – 89	32
70 – 79	43
60 – 69	21
50 – 59	11
40 – 49	3
30 – 39	1

Find,

(i) The quartiles of the distribution.

(ii) The mean mark of the distribution. (Use assumed mean = 64.5)

(b) In an agricultural experiment, seeds were planted in rows and the number of seeds that germinated in each row was tallied. The data are summarized in table shown.

Number of seeds germinated per row	Frequency of rows
0 – 4	20
4 – 8	45
8 – 12	30
12 – 16	5

Find the standard deviation of seeds germinating per row. Give your answer correct to four decimal places.

(10 marks)

5. (a) Using the laws of algebra of sets, simplify each of the following expression:

(i) $(P \cap Q') \cup (P \cup Q')$

(ii) $(P \cap Q') \cup (P' \cap Q) \cup (P \cap Q)$

(b) Given that

$$\mu = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$$
$$A = \{10, 12, 13, 15, 18, 19\}$$
$$B = \{11, 12, 14, 15, 17\}$$

Find (i) $n(A' \cup B)'$, (ii) $A' \cup (A \cap B')$

(c) In a class of thirty pupils, eighteen are taking Geography and of these eleven are taking both Geography and Economics. There are five pupils in the class who take neither Geography nor Economics. How many pupils in this class who take Economics? (Use Venn diagram)

(10 marks)

6. (a) Given the functions $f(x) = x^2 + 5$ and $g(x) = \sqrt{x+1}$,find:

(i) $g \circ f(-3)$,

(ii) Sketch the graph of $g \circ f(x)$ and state its domain and range.

(b) If $f(x) = \dfrac{2x-3}{x^2+2x-3}$, find the vertical asymptotes and hence sketch the graph of $f(x)$.

(10 marks)

7. (a) Use the Trapezoidal and the Simpson's rules with eleven ordinates to find approximate value of $\int_0^1 \frac{dx}{2+2x^2}$. Compare your results with exact value of the integral and hence state which rule is more correct. (Give your answers correct to four decimal places).

(b) Derive the Secant formula.

(c) The equation $x^3 - 3x - 20 = 0$ has a single real root inside the interval $[3, 4]$. Approximate the root in four iterations using the secant formula obtained.

(10 marks)

8. (a) (i) Given that θ_1 and θ_2 are the angles that lines L_1 and L_2 make with the x – axis respectively, derive the formula to find the angle between L_1 and L_2 where $\theta_1 > \theta_2$.

(ii) Use the formula obtained in part (a) (i) to find the acute angle between $4x - 3y - 5 = 0$ and $2x + y - 1 = 0$.

(b) Find the perpendicular distance of the point $(6, 8)$ from the line $y = 3 - \frac{5}{4}x$.

(c) A point P moves so that it is equidistant from the points A(1, 2) and $B(-2, -1)$. Find the cartesian equation of the locus of P.

(10 marks)

9. (a) Evaluate $\int \frac{3\,dx}{15 + 9\cos x}$.

(b) Evaluate $\int_0^{\frac{\sqrt{2}}{2}} \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$.

(c) Find the length of the arc of the parabola $y = x^2$ from $x = 0$ to $x = 1$.

(10 marks)

10. (a) Differentiate $x^2 \cos^{-1}\left(\frac{x}{2}\right)$ with respect to x.

(b) Find $\frac{dy}{dx}$ if $y = \frac{e^{2x} \ln x}{(x-1)^3}$.

(c) Use Maclaurin's theorem to expand $\ln(2 + x)$ in ascending powers of x as far as the term in x^5.

(10 marks)