THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 **ADVANCED MATHEMATICS 1**
(For Both School and Private Candidates)

Time: 3 Hours **Monday, 05th May 2014 a.m.**

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

1. Using a non-programmable calculator:

(a) (i) Evaluate $\dfrac{6.2\ln\sqrt{7}\div\ln\sqrt{3}}{1782\log 1783}$ and write your answer to six significant figures.

(ii) Compute $\dfrac{3^{1.75}\times\log_2 14}{\tan^{-1}(3.42)\times\log_e 13.27}$ to seven significant figures.

(b) The volume of a tetrahedron is given by $v=\dfrac{1}{6}a^3(1-\cos\theta)(1+2\cos\theta)^{\frac{1}{2}}$ where a is the length of the edges and θ an angle made by the edges. By completing the table below, find the volume of the tetrahedron for the given values of a and θ and write your answers correct to three decimal places.

a	θ	$\cos\theta$	$2\sqrt{\dfrac{1+\cos 2\theta}{2}}$	Volume (v) in cubic unit
1 unit	$\dfrac{\pi}{12}$			
2 unit	$\dfrac{\pi}{9}$			
3 unit	$\dfrac{\pi}{6}$			

2. (a) (i) Sketch the graphs of the functions $y=\cosh x$ and $y=\sinh x$ on the same x – y plane.

(ii) Using part (a) (i), state the range of $\cosh x$ and $\sinh x$.

(b) (i) Prove that $\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$.

(ii) Solve the equation $3\operatorname{sech}^2 x+4\tanh x+1=0$ and write your answer correct to 4 decimal places.

(iii) Verify that $\sinh 3x=3\sinh x+4\sinh^3 x$.

(c) Show that $\dfrac{1}{e^{-3\ln x}}\left(\sqrt{\dfrac{\cosh\ln x-\sinh\ln x}{\cosh\ln x+\sinh\ln x}}\right)=x^2$.

3. (a) A farm is to be planted with cabbages and potatoes. The cost and the number of people needed for the work is indicated in the table below:

	Cabbages	Potatoes	Total available
Labour per hectare (Number of people)	2	1	10
Labour costs per hectare (Tshs)	28,000/=	24,000/=	168,000/=
Costs of fertilizer per hectare (Tshs)	60,000/=	80,000/=	480,000/=

(i) Find the greatest number of hectares that can be planted.

(ii) If the profit for a hectare of cabbages is 80,000/= and for potatoes is 60,000/=, how many hectares of each crop should be planted to maximize the profit?

(b) One of the Tanzanian wine drink manufacturing firms has m plants located in different towns. The total production is absorbed by n retail shops in different towns.

(i) Formulate the general transporting schedule that minimizes the total cost of transporting wine drinks from various plants to various retail shops.

(ii) Construct the transportation table with 2 origins and 2 destinations using the following parameters: The Supply is a_i, demand b_j and the cost c_{ij}.

(iii) From (b) (i) and (ii), deduce the conditions which satisfy the existence of a feasible solution to the transportation problem.

4. (a) The monthly wages of employees working in a certain factory are given in the table below:

Wages in shs (× 1000)	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100	100 - 110	110 - 120
Number of employees	8	10	16	15	10	8	3

(i) By using an appropriate formula, find the median and mode for the wages given above, giving your answer to the nearest thousand shillings.

(ii) Find the semi-interquartile range of the given data.

(b) The number of errors made by the typist on each page of a document with 100 pages were recorded in the table below.

Number of errors	Frequency
0	15
1	30
2	28
3	18
4	9

(i) Find the variance and standard deviation of the number of errors per page, writing your answer correct to 4 decimal places.

(ii) Find the 20^{th} percentile of the data.

5. (a) Using Venn diagrams show that:
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(b) By using set properties, prove that for any non-empty sets X and Y:

(i) $X \cup (X \cap Y) = X$,

(ii) $(X \cap Y') \cup (Y \cap X') = (X \cup Y) - (X \cap Y)$.

(c) There are twenty five men at a meeting of which eleven are doctors, sixteen are teachers, and eight are both doctors and teachers. How many are neither doctors nor teachers?

6. (a) (i) A function is defined by $g : x \to x^2 - 10$ for all $x \in \Re$. Find all the values of x for which $g \circ g(x) = 26$.

(ii) If $f(x) = 3x - 2$, $g(x) = x + 7$ and $h(x) = \dfrac{1}{1+x}$, determine the intercepts and the asymptotes of $f \circ g \circ h$.

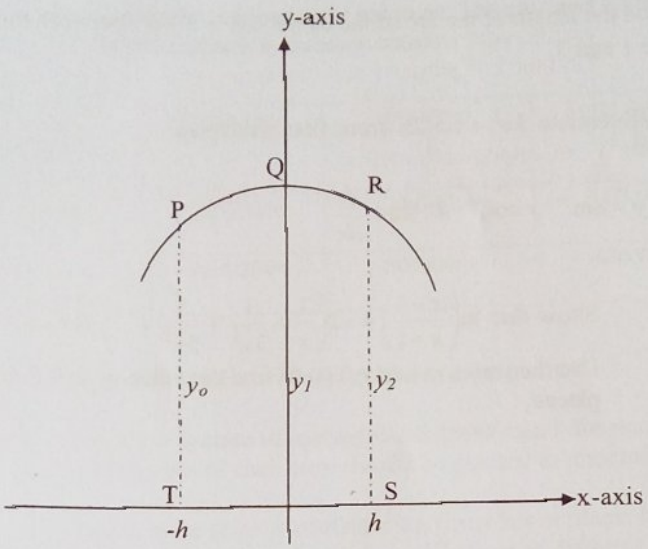
(b) Given that $f(x) = x^4 - 2x^3 - x^2 + 2x$,

(i) Find the value of x where the curve $f(x)$ cuts the x – axis.

(ii) Sketch the graph of $f(x)$.

7. (a) (i) Apply the Newton – Raphson formula with three iterations to compute the value of $\sqrt[3]{7}$ correct to five significant figures. Use $x_0 = 2$.

(ii) The figure below has points P, Q and R on the quadratic curve $f(x) = ax^2 + bx + c$. Derive the Simpson's rule with n-ordinates to approximate the area PQRST.



(b) (i) Evaluate $\int_0^{\frac{\pi}{4}} \cos^2 x dx$ by using the Simpson's rule with five ordinates and write your answer to 4 decimal places.

(ii) Find the actual value of $\int_0^{\frac{\pi}{4}} \cos^2 x dx$ and compare your answers with part (b) (i).

8. (a) Sketch the diagram for the locus of points which move such that it covers a distance a units from the curve $x^2 + y^2 + 2x - 4y = 20$ where $|a| < 5$.

(b) Find the length of the tangent from the point (5, 7) to the circle $x^2 + y^2 - 4x + 6y + 9 = 0$.

(c) If p and q are the lengths of the perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\operatorname{cosec}\theta = k$ respectively, prove that $p^2 + 4q^2 = k^2$.

9. (a) If the gradient of a certain function is $\frac{1}{7(e^x + 1)}$, find the function.

(b) Evaluate the following integrals:

(i) $\int_1^5 \frac{2t}{\sqrt{2t+1}}$ (leave your answer in surd form).

(ii) $\left| \int_0^{\frac{\pi}{2}} \cos 2x \sin x dx \right|$.

(c) Find the length of the arc of the curve $6xy = 3 + x^4$ between the points whose abscissa are 1 and 3.

10. (a) Differentiate $3x^2 + \cos 2x$ from first principles.

(b) If $y = \sin^{n+1} x \cos^{m-1} x$, find $\frac{dy}{dx}$.

(c) (i) Show that $\ln\left(\frac{x-1}{x+1}\right) = -2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \ldots\right)$ for $|x| > 1$.

(ii) Use the series in part (c) (i) to find the value of $\ln 0.5$ correct to three decimal places.