THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours **Monday, 02nd May 2016 a.m.**

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. **All** necessary working and answers for each question done must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

1. (a) Using a scientific calculator find the following correct to four decimal places:

(i) $$\frac{\sqrt{(3.12\times\log 5)^3}}{\sqrt[3]{\left(\cos\frac{\pi}{9}+\sin 46^\circ\right)}}.$$

(ii) $$\left[\frac{\sqrt{e^3\log_2 6}\times\sinh^{-1}(0.6972)}{(\ln 3.5)\times(\cos 64.5^\circ)\times(\tan 46^\circ)}\right]\times(0.6467)^3.$$

(b) A rat has a mass 30 grams at birth. It reaches maturity in 3 months. The rate of growth is modeled by the equation $\frac{dm}{dt}=120\,(2.1985t-3)^2$, where m grams is the mass of the rat, t months after birth. Use the scientific calculator to find the mass of the rat when fully grown.

2. (a) If $t=\tanh\frac{x}{2}$, express $\sinh x$ and $\cosh x$ in terms of t.

(b) Express $\sinh^{-1}x-\ln x$ in terms of natural logarithms; hence, find the limit as $x\to\infty$.

(c) Evaluate $\int_4^7\frac{1}{\sqrt{(4x^2-8x+7)}}dx$ correct to four decimal places.

3. (a) Mr. Mutu takes two types of vitamin pills. He must have at least 16 units of vitamin A, 5 units of vitamin B and 20 units of vitamin C. He can choose between pill M which contains 8 units of A, 1 unit of B and 2 units of C; and pill N which contains 2 units of A, 1 unit of B and 7 units of C. Pill M costs 150 shillings and pill N costs 300 shillings. How many pills of each type should he buy in order to minimize the cost?

(b) A TV dealer has stores in Dar es Salaam and Morogoro and retailers in Tanga and Dodoma. The stores have a stock of 45 TV and 40 TV sets respectively while the requirements of the retailers are 25 and 30 TV sets respectively. If the cost of transporting a TV set from Dar es Salaam to Tanga is Tsh 5,000/= and from Dar es Salaam to Dodoma is Tsh 9,000/=, from Morogoro to Dodoma is Tsh 3,000/= and Morogoro to Tanga is Tsh 6,000/=;

(i) How should the TV dealer supply the requested TV sets at minimum cost?

(ii) What is the minimum cost?

4. (a) The frequency distribution of a variable X is classified in equal intervals of size C. The frequency in a class is denoted by f and the total frequencies is N. If the data is coded into a variable u by means of the relation $\bar{x} = a + C\bar{u}$, where X takes the central values of the class intervals, show that the standard deviation δ of the distribution is given by $\delta^2 = C^2\left(\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2\right)$

(b) The average heights of 20 boys and 30 girls are 160 cm and 155 cm respectively. If the corresponding standard deviations of boys and girls are 4 cm and 3.5 cm, find the standard deviation of the whole group.

(c) The following table shows the length of 100 earth worms in millimetres:

Length(mm)	95 - 109	110 - 124	125 - 139	140 - 154	155 - 169	170 - 184	185 - 199	200 - 214
Number of worms	2	8	17	26	14	16	6	1

Obtain the semi-interquartile range correct to two significant figures.

5. (a) Use the laws of algebra of sets to;

(i) verify that $X \cup (X \cap Y) = X$.

(ii) simplify $\left[A \cap (A \cup B)'\right]'$.

(b) If A, B and C are three non-empty sets, use venn diagram to show whether $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

(c) A class contains 15 boys and 15 girls. A survey of the class showed that;

20 pupils were studying Geography,
14 pupils were studying Mathematics,
10 of the girls were studying Geography,
4 of the girls were studying Mathematics,
3 of the girls were studying both Geography and Mathematics,
3 of the boys were studying neither Geography nor Mathematics.

How many pupils were studying both Mathematics and Geography? (Use Venn Diagrams).

6. (a) Use the table of values to draw the graph of $f(x)=2+e^{2x}$ if $-3\le x\le 1.2$ and $g(x)=1-e^{x}$ if $-3\le x\le 2.7$ on the same xy plane.

(b) Given that, $f(x)=x+1-\frac{1}{x}$ and that $g(x)=\frac{1}{x}$;

(i) Write down the composite function $g\,o\,f(x)$ in its simplest form.

(ii) Find the value of x if $g\,o\,f(x)=f\,o\,g(x)$.

(c) Find the equation of the asymptotes of the curve $y=\frac{x^2+3}{x-1}$ and sketch the curve showing the coordinates of the turning points.

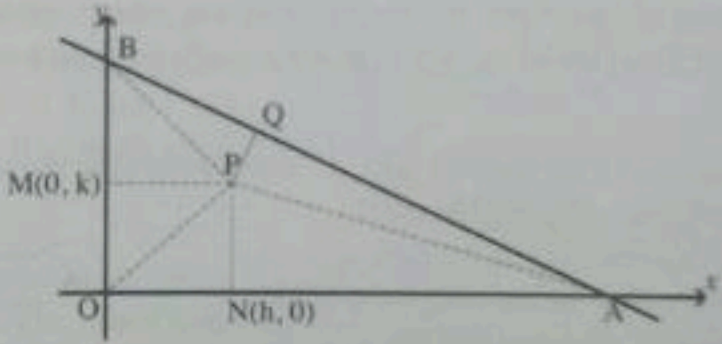
7. (a) (i) Write down four sources of errors in numerical computations.

(ii) If x_{n+1} is a better approximation to a root of the equation $f(x_n)=0$. Derive the Newton-Raphson method for the function $f(x_n)$.

(b) Use the Newton-Raphson method obtained in (a) (ii) to derive the secant formula and comment why would you want to use it instead of the Newton-Raphson method.

(c) Using the secant method obtained in (b) with $x_1=2$ and $x_2=3$ perform three iterations to approximate the root of $x^2-2x-1=0$ and hence compute the absolute error correct to four decimal places.

8. (a) (i) The line $Ax+By+C=0$ meets coordinates axes at A and B. If P is a point (h,k) and $PQ=p$ is the perpendicular distance to AB. Use the information given and the figure below to derive the perpendicular distance of the point P from the line AB.



(ii) The perpendicular distance from the point $(2,5)$ to the line $ny=2x-4$ is $\sqrt{5}$. Find the value of n.

(b) Write down the equation to the bisector of the acute angle between the lines $3x+4y=1$ and $5x-12y+6=0$.

(c) Find the length of a tangent from the centre of the circle $x^2+y^2+6x+8y-1=0$ to the circle $x^2+y^2-2x+4y-3=0$.

9. (a) (i) Show whether $\int \frac{f'(x)}{f(x)}dx = \ln A f(x)$, where A is a constant.

(ii) Find $\int \cos 2x \cos 4x \cos 6x \, dx$.

(b) Evaluate $\int_0^{\frac{\pi}{2}} x \sin x \cos x \, dx$.

(c) Find the area of the region bounded by the curve $y=3x^2-2x+1$, the lines $x+1=0$, $x-2=0$ and $y=0$.

(d) The area between the curve $3x^2+y^2=9$ and the y – axis from $y=-3$ to $y=3$ is rotated about the y-axis. Find the volume of the solid generated.

10. (a) Find the derivative of $\frac{1}{x}+\cos 3x$ from first principle.

(b) Use the Taylor theorem to obtain the series expansion for $\cos\left(x+\frac{\pi}{3}\right)$ stating terms including that in x^3. Hence obtain a value for $\cos 61°$ giving your answer correct to five decimal places.

(c) Show whether the line $2x-y=0$ and the curve $4x^2-4xy+y^2 4x-8y+10=0$ intersect at a right angle.

(d) A two variable function f is defined by $z=f(x,y)=x^2+xy+y^2$; find $\frac{\partial z}{\partial y}$ at $(1,1,1)$.