THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION
EXAMINATION

142/2

ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **Thursday, 08th May 2014 a.m.**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

SECTION A (60 marks)

Answer **all** questions in this section.

1. (a) Obtain the five roots of the equation $z^5 = 32$.

(b) If $Q(x, y)$ is a point on the Argand diagram corresponding to $z = x + iy$ and $|z+2| = 4|z-2+3i|$, find the Cartesian equation of the locus of Q.

(c) Prove that $\tan 5\theta = \dfrac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$.

(d) If $Z = x + iy$ and $\overline{Z}$ is a conjugate of Z, find the values of x and y such that $\left(\dfrac{1}{Z}\right)^{-1} + \left(\dfrac{2}{\overline{Z}}\right)^{-1} = 1 + i$.

(15 marks)

2. (a) Show whether the following statement is valid or not:
If I study hard, then I will not fail Geography. If I do not fail to manage my time then I will study hard. But I failed Geography. Therefore I failed to manage my time.

(b) Let p be 'He is tall' and q be 'He is handsome' write each of the following statements in symbolic form using p and q:
(i) He is tall and handsome.
(ii) He is neither tall nor handsome.

(c) (i) Simplify the compound statement $q \vee (p \wedge q') \vee (r \wedge p')$ using the algebraic laws for logical expressions and then draw the corresponding network.
(ii) Construct the compound sentences for S_1 and S_2 having the following truth table and simplify S_1 using algebraic laws for logical expressions.

p	q	r	S_1	S_2
T	T	T	F	T
T	T	F	F	F
T	F	T	T	F
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	T	F
F	F	F	F	F

(d) Use a truth table to show that the statement $\sim P \wedge [Q \wedge (\sim Q \vee P)]$ is a self-contradiction.

(15 marks)

3. (a) Forces of magnitude 5 and 7 units acting in the direction of $2\underline{i}+3\underline{j}+2\underline{k}$ and $3\underline{i}+2\underline{j}+2\underline{k}$ respectively act on a particle which is displaced from the point $(2, 2, -1)$ to $(4, 3, 1)$. Find the work done by the forces.

(b) A particle with 200g of mass is moving along a curve with the velocity $\frac{i}{4t}-\frac{7j}{t}+\frac{k}{1-2t}$.

(i) Find the force applied to the particle at any time t.

(ii) Find the position vector at time t where the particle is heading.

(c) Find the area of a parallelogram having the diagonals $\underline{A}=3\underline{i}+\underline{j}-2\underline{k}$ and $\underline{B}=\underline{i}-3\underline{j}+4\underline{k}$ (leave your answer in surd form).

(d) Use the definition of dot product to prove the law of cosine for a triangle ABC.

(15 marks)

4. (a) (i) Find the adjoint of $\begin{pmatrix} 3 & -2 & -2 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$.

(ii) Use the adjoint obtained in 4 (a) (i) to solve the system of equations

$$3x-2y-2z=1$$
$$2x+3y-z=13$$
$$x-y+3z=-8$$

(b) If x is so small that x^3 and higher powers may be neglected, obtain a quadratic approximation of $f(x)=\frac{1+e^x}{1+\ln(1+x)}$.

(c) Express $\frac{2x+5}{x^2+3x+2}$ in partial fractions and hence state the first four terms in the series expansion of $\frac{2x+5}{x^2+3x+2}$.

(d) Write the following series in the sigma notation:

(i) $\frac{2}{7}-\frac{4}{7}+\frac{8}{7}-\frac{16}{7}+\ldots\ldots\ldots+\frac{128}{7}$,

(ii) $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots\ldots\ldots+\frac{1}{42}$.

(15 marks

SECTION B (40 marks)

Answer any **two (2)** questions from this section. Extra questions will not be marked.

5. (a) Show that $2\tan^{-1}2 + \tan^{-1}3 = \pi + \tan^{-1}\frac{1}{3}$.

(b) Find all angles which satisfy the equation $10\sec^2\theta - 3 = 17\tan\theta$.

(c) (i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$.

(ii) By using the results in part (c)(i) express $\cos^2 A\left(16\cos^4 A - 24\cos^2 A + 9\right)$ as a single term of cosine.

(iii) If α, β and γ are the angles of a triangle, prove that

$$\cos\alpha + \cos\beta + \cos\gamma - 1 = 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}.$$

(d) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta - y\cos\theta = 0$, prove that $x^2 + y^2 = 1$.

(20 marks)

6. (a) The random variable X has the following probability table:

Value of X	0	1	2
Probability of the value of X	k	2k	3k

(i) Determine the value of the constant k.

(ii) Find $E(X)$.

(b) If a random variable X has probability density function

$$f(x) = \begin{cases} kx, & 0 \le x \le 2 \\ k(4-x), & 2 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of the constant k.

(ii) Evaluate $P\left(\frac{1}{2} \le X \le \frac{5}{2}\right)$.

(c) In a class of 20 students, each has 45% chance to pass CSEE 2014. Find the probability that;

(i) Exactly 15 students will pass.

(ii) At least two students will pass.

(iii) More than 17 students will pass.

(Write your answers to six decimal places).

(d) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 20 tools chosen at random exactly 5 will be defective, by using

(i) The binomial distribution.

(ii) Poisson approximation to the binomial distribution.

(Write your answers to four decimal places).

(20 marks)

7. (a) Obtain the first order differential equation of $y = cx^2 + c^2$.

(b) Find the general solution of the differential equation $\frac{dy}{dx} = \frac{2x + y - 2}{2x + y + 1}$.

(c) Find a curve on the x-y plane that passes through the origin and for which the tangent at (x, y) has slope $x^2 + y$.

(d) Solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 2\cos t - 4\sin t$ given the initial conditions $x = 2$ and $\frac{dx}{dt} = -3$ at $t = 0$.

(e) Determine whether or not the following functions are solutions of the differential equation $\frac{d^2y}{dx^2} - y = 0$. (i) $y = 2\sin x$ (ii) $y = e^{-2x}$ and (iii) $y = \frac{4}{e^x}$.

(20 marks)

8. (a) (i) Find the equation of the normal to the parabola $\frac{y^2}{16x} = p$ at the parametric coordinates $\left(\frac{pt^4}{4}, 2pt^2\right)$ where p is constant.

(ii) Find the equation of the tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the parametric coordinates $(3\cos\theta, 2\sin\theta)$.

(b) (i) Derive the equations of asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(ii) Find the eccentricity and foci of the curve $\frac{(x-2)^2}{4} - \frac{(y-3)^2}{9} = 1$.

(c) Prove that the equation $r = \frac{4}{1 + \cos\theta}$ represents a translated parabola.

(d) Given the equation $y + \frac{x^2 - 10x + 25}{12} - 4 = 0$:

(i) Write the equation in the standard form of the parabola.
(ii) Find the line of symmetry of the parabola in (d) (i).
(iii) Find the focus of the parabola in (d) (i).

(20 marks)