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THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Time: 3 Hours **Thursday, 07th May 2015 a.m.**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) Use the Euler formula for exponentials $z=e^{i\theta}$, to show that

$$\frac{1}{2}\left(z+\frac{1}{z}\right)=1-\frac{\theta^2}{2}-\frac{i\theta^3}{6}+\frac{\theta^4}{24}+\dots$$

(b) Given that one root of the equation $z^4+z^3+3z^2+z+2=0$ is i, find the other roots.

(c) (i) If $z_1=1+i\sqrt{3}$ and $z_2=\sqrt{3}+i$, find the modulus and the principle argument of z_1z_2.

(ii) If z is a complex number find the equation of locus represented by $|z-(2-i)|=|z-(3+2i)|$.

(d) If $z=\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ and $z^2w=21\left[\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right]$, express w in modulus argument form.

(15 marks)

2. (a) (i) Prepare a truth table for the proposition $((q\rightarrow\sim p)\wedge((p\vee r)\wedge q))\rightarrow r$.

(ii) Determine the truth value and comment on the validity of the argument below using truth table.

$p\rightarrow(q\vee\sim r)$
$q\rightarrow(p\wedge r)$
$\overline{[(q\vee\sim r)\wedge(p\wedge r)]\rightarrow r}$

(b) Using the laws of algebra in logic:

(i) Determine the validity of the following argument: "If there is rain, the crops will grow well. If crops grow well, there is no famine. But there is famine. Therefore there is no rain.

(ii) Simplify the proposition $\sim((p\vee q)\vee(\sim p\wedge q))$.

(c) Translate the following compound statements in symbolic notation using letters P, Q and R to stand for the statements:

(i) Either the manufactured drug is not fault and accepted by the Tanzania Food and Drug Authority (TFDA) or the drug is fault and is not accepted by the TFDA.

(ii) If Kapirima is a member of a social committee then the committee is strong. The committee is strong if and only if Kaprima's argument is accepted by other members. Therefore Kaprima's argument is not accepted and the committee is not strong.

(15 marks)

3. (a) (i) Find the expression for the work done used to move a 15kg baby from $\underline{r} = u_1\underline{i} + u_2\underline{j}$ to $\underline{p} = v_1\underline{i} + v_2\underline{j}$ and hence deduce the actual work done when (u_1, u_2) and (v_1, v_2) are $(-1,7)$ and $(2,3)$ respectively (use $g = 9.8$).

(ii) Two forces of 40N and 60N act on a point in a plane. If the angle between the force vectors is 30°, find the magnitude and direction of the resultant force relative to the 60N force (write the magnitude of the resultant force to two significant figures and the angle to the nearest degree).

(b) The vertices of a triangle are A (1, 1, 2), B (3, 2, -1) and C (-4, 1, 3). Use your knowledge on vectors to find the area of triangle ABC.

(c) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$ where t is time. If its position vector is $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$;

(i) Determine its velocity and acceleration at any time t.

(ii) Find the magnitude of the velocity and acceleration at time $t = 0$.

(15 marks)

4. (a) Solve the following system of equations by using Cramer's rule.

$$2x + y - z = 3$$
$$x - y + z = 0$$
$$x + 2y + z = -3$$

(b) (i) Use binomial expansion of $\left(1 - \frac{1}{50}\right)^{\frac{1}{2}}$ to find the value of $\sqrt{2}$ correct to seven significant figures.

(ii) Decompose $\frac{2}{4n^2 - 1}$ into partial fractions and hence find $\sum_{n=1}^{n} \frac{2}{4n^2 - 1}$.

(c) One of the zeros of the polynomial function $f(x) = x^4 - (2+h)x^3 + (2h-5)x^2 + (5h+6)x - 6h$ is obtained when $h = 1$. Find the value of the constants p, q and r when $f(x) = (1 - 2x + x^2)(px^2 - qx - r)$.

(d) Given the simultaneous equations $\begin{cases} 3^x - 2^y = 0 \\ x + y - 1 = 0 \end{cases}$ show that $y = \log_6 3$.

(15 marks)

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$(1 - 0.02)^{0.5}$

SECTION B (40 Marks)

Answer any **two (2)** questions from this section. Extra questions will not be marked.

5. (a) Solve the trigonometric equation $\sec^2\theta + \tan\theta - 1 = 0$ for $0° \leq \theta \leq 360°$.

   (b) Factorize completely the trigonometric expression $\cos a - \cos 3a - \cos 5a + \cos 7a$.

   (c) (i) Verify that $\dfrac{\cos^2 t - 3\cos t + 2}{\sin^2 t} = \dfrac{2 - \cos t}{1 + \cos t}$

   (ii) Prove that $\dfrac{\sin 3A \sin 6A + \sin A \sin 2A}{\sin 3A \cos 6A + \sin A \cos 2A} = \tan 5A$.

   (d) Show that $\dfrac{\sin^2 x + 2\sin x + 1}{\cos^2 x} = \dfrac{\cos^2 x}{1 - 2\sin x + \sin^2 x}$

(20 marks)

6. (a) A school needs 10 prefects out of which 5 are supposed to be girls and 5 are to be boys. If 5 boys are to be selected from a group of 8 boys and 5 girls from 9 girls; in how many different ways can the 10 prefects be selected.

   (b) Three athletes from Tanzania will participate in an International Coca Cola marathon race next year. If the probabilities to complete the marathon are 0.9, 0.7 and 0.6 respectively, find the probability that at least two of them will complete the marathon.

   (c) If a random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{|x|}{8} & \text{for } -2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

   Calculate the
   (i) expected value of X.
   (ii) Standard Deviation of X.
   (iii) Variance of X.

   (d) If $P(A \cup B) = 80\%$ and $P(A \cup B') = 70\%$ determine $P(A)$.

(20 marks)

7. (a) (i) Form the first order differential equation which represent the family of the curve $x^2 + y^2 - 2kx = 0$.

   (ii) Find the particular solution of the differential equation $x\dfrac{dy}{dx} = x + y$, given that $y = -1$ when $x = 1$.

   (b) Solve the initial value problem $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x$, $y(0) = 4$ and $\dfrac{dy}{dx} = -8$ when $x = 0$.

(c) The rate of decrease of temperature of water is direct proportional to the difference between temperature of water and that of the medium. If water at a temperature of $100C^{\circ}$ cools in 10 minutes to $80C^{\circ}$ in a room temperature of $25C^{\circ}$ find:

(i) temperature of water after 20 minutes.

(ii) the time when the temperature is $40C^{\circ}$.

(Any approximation in calculations must be presented in 5 decimal places)

(d) If $\frac{dy}{dx}=\frac{x^2y}{x^3+1}$, find the equation of the solution curve which passes through $(2,3)$.

(20 marks)

8. (a) (i) Find the equation of the tangent to the ellipse $9x^2+25y^2-18x-100y-116=0$ at $(1,5)$.

(ii) The normal at the point $p(4\cos\theta, 3\sin\theta)$ on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ meets the x-axis and y-axis at A and B respectively. Show that the locus of the midpoint of AB is an ellipse with the same eccentricity as the given ellipse.

(b) (i) Determine the polar equation of $x^2+y^2-2x-3y=0$.

(ii) Draw the graph of the polar equation obtained in part b (i) above.

(c) (i) Show that the line $3x-y+1=0$ touches the parabola $y^2=12x$.

(ii) If $ax+by+c=0$ touches the parabola $x^2-8y=0$, find an equation connecting a, b, and c.

(20 marks)

$r\cos\theta\, e\varphi = r$