THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours **Thursday, 05th May 2016 a.m.**

Instructions

1. This paper consists of **eight (8)** questions in sections A and B.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) (i) Use the De moivre's theorem to find the value of $(1+i)^5$.

(ii) Use the mathematical induction to prove that

$(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + \sin n\theta)$.

(b) (i) If $\arg\left(\frac{z-1}{z+i}\right) = \frac{\pi}{4}$ and $z = x+iy$, find the locus of the point representing z in an argand diagram.

(ii) Solve the following system of equations where z and w are complex numbers.

$$\begin{cases} iz - w = 2i \\ iz + iw = i \end{cases}$$

(c) One of the roots of the equation $z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$ is $1+i$. Find the other roots. **(15 marks)**

2. (a) (i) Draw the simplified electrical circuit for the argument:

$[p \wedge (p \vee q)] \vee [q \wedge \sim (p \wedge q)]$

(ii) Use the truth table values only to show whether or not $p \leftrightarrow \sim q$ and $\sim (p \wedge q)$ are logically equivalent.

(b) (i) Use the truth table to test the validity of the following argument: If I am intelligent, then I will pass this examination. I am intelligent. Therefore I will pass this examination.

(ii) Write the converse, inverse and contrapositive of the statement 'If Mathematics is interesting then Biology is boring and tough'. **(15 marks)**

3. (a) If the position vector $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ are defined by $\overline{OA} = 2\underline{i} - \underline{j} + 3\underline{k}$, $\overline{OB} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ and $\overline{OC} = -\underline{i} + 3\underline{j} - 2\underline{k}$;

(i) Determine the cross product $\overline{AB} \times \overline{BC}$.

(ii) Find the exact value of the angle between $\overline{AB}$ and $\overline{BC}$.

(b) If $\underline{a} = 3\underline{i} + 2\underline{j} + 9\underline{k}$ and $\underline{b} = \underline{i} + \lambda\underline{j} + 3\underline{k}$. Find

(i) The value of λ so that $\underline{a} + \underline{b}$ is perpendicular to $\underline{a} - \underline{b}$.

(ii) The projection of $\underline{a}$ onto $\underline{b}$ and leave the answer in surd form.

(c) Derive the cosine's rule using the vectors $\underline{u}$ and $\underline{v}$. **(15 marks)**

4. (a) (i) Find the value of a if the 17^{th} and 18^{th} terms of the expansion $(2+a)^{60}$ are equal.

(ii) The roots of the equation $x^3 + px^2 + qx + 30 = 0$ are in the ratio 2:3:5. Find the value of p and q.

(b) (i) State the principle of Mathematical Induction as it is used in mathematics.

(ii) Use the principle of mathematical induction to prove that $\sum_{r=1}^{n} 3r - 1 = \frac{n}{2}(3n+1)$

(15 marks)

SECTION B (40 Marks)

Answer any **two (2)** questions from this section. Extra questions will not be marked.

5. (a) (i) Solve the equation $\tan^{-1}\left(\frac{x-1}{x+2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ and leave the answer in surd form.

(ii) Prove the identity $\frac{1+\sin x}{1-\sin x} \equiv (\tan x + \sec x)^2$.

(b) If $2\sin\theta + \cos\theta = 1$, use t-formula to find the value of θ in the interval $0° \le \theta \le 180°$.

(c) (i) Show that $\frac{\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot\left(\frac{5\theta}{2}\right)$.

(ii) Verify that $\frac{\sin(A+B+C) + \sin(A-B-C)}{\cos(A+B+C) - \cos(A-B-C)} = \frac{\tan B \tan C - 1}{\tan B + \tan C}$.

(d) Express $3\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$ giving values of R and α. **(20 marks)**

6. (a) The probability that a keyboard picked at random from the assembly line in a factory will be defective is 0.01. If a sample of three is to be selected:

(i) Construct the probability distribution of the defective keyboards.

(ii) Find the mean and standard deviation (Give your answers correct to 2 decimal places).

(b) The bag R contains 5 red and 3 green balls and bag P contains 3 red and 5 green balls. If one ball is drawn from bag R and two from bag P, find the probability that out of three balls drawn two are red and one is green.

(c) The random variable X has a probability distribution $P(x)$ of the following form, where k is a certain number.

$$P(X) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Determine the value of k.
(ii) Find $P(x<2)$, $P(x\leq 2)$ and $P(x\geq 2)$.

(d) (i) If X is a discrete random variable where $E(X)$ is the expected value of X, show that $E(Ax+b)=aE(x)+b$ where a and b are constants.
(ii) The modern seeds of a certain crop have the probability of germinating 0.9. If six seeds are sown, what is the probability of at most 5 seeds are germinating?
(20 marks)

7. (a) (i) If $x(1-y)\frac{dy}{dx}+2y=0$ and $y=2$ when $x=e$, show that $x^2ye^{-y}=2$.

(ii) Solve the differential equation $(2x-y)\frac{dy}{dx}=2x-y+2$ given that $y=1$ when $x=2$.

(b) Form a differential equation whose solution is the function $y=Ae^{2x}+Be^{-3x}$ where A and B are arbitrary constants.

(c) A tank contains a solution of salt in water. Initially the tank contains 1000 litres of water with 10 kg of salt dissolved in it. The mixture is poured off at a rate of 20 litres per minute, and simultaneously pure water is added at a rate of 20litres per minute. All the time the tank is stirred to keep the mixture uniform.
(i) Find the mass of the salt in the tank after 5 minutes.
(ii) How long the mass of the salt in the tank falls to 5kg?

(d) Find the general solution of the differential equation $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=10e^{-2x}$.
(20 marks)

8. (a) (i) If $9y^2-54y-25x^2+200x-544=0$ is the hyperbola equation; find the center, the vertices, the foci and the equation of the asymptotes.
(ii) Convert $x^2+y^2=4x$ into polar equation.
(iii) Convert $(1,-1)$ into polar coordinates.

(b) Find the equation of the tangent and normal at $P(a\cos\alpha, b\sin\alpha)$ to the ellipse $b^2x^2+a^2y^2=a^2b^2$.

(c) (i) Define a conic section.
(ii) A man running a race- course notes that the sum of the distances from the two flag posts to him is always 10 meters. If the distance between the flag posts is 8 meters, find the equation of the path traced by the man.

(d) Sketch the graph of $r=2(1+\sin t)$. **(20 marks)**