THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2

(For Both School and Private Candidates)

Time: 3 Hours **Tuesday, 09th May 2017 p.m.**

Instructions

1. This paper consists of **eight (8)** questions in sections A and B.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) Use the Demoivre's theorem to find the value of $\left(\frac{1}{2}+\frac{1}{2}i\right)^{10}$.

(b) Show that $\left[r(\cos\theta+i\sin\theta)\right]^n = r^n e^{in\theta}$ and hence find in form of $re^{i\theta}$ all complex numbers z, such that $z^3 = \frac{5+i}{2+3i}$.

(c) (i) Solve the equation $x^4+1=0$ and leave the roots in radical form.

(ii) If $w=\frac{z+2}{2}$ and $|z|=4$, find the locus of the w. **(15 marks)**

2. (a) (i) Write the contrapositive of the inverse $p \rightarrow q$.

(ii) Use the truth table to verify that the statement $(p \vee q)\wedge((\sim p)\wedge(\sim q))$ is a contradiction.

(b) (i) Use the laws of algebra of propositions to simplify the statement $q \vee (p \wedge \sim q) \vee (r \wedge q)$ and hence draw the corresponding simple electrical network.

(ii) Use the truth table to show that $p \leftrightarrow q$ logically implies $p \rightarrow q$.

(c) Without using the truth tables, prove that the proposition $[(p \rightarrow q)\wedge(\sim q)] \rightarrow \sim p$ is tautology. **(15 marks)**

3. (a) (i) If $\underline{a} = 3\underline{i} - 5\underline{j} - 2\underline{k}$ and $\underline{b} = 7\underline{i} + \underline{j} - 2\underline{k}$ are non-zero vectors. Find the projection of $\underline{a}$ onto $\underline{b}$.

(ii) Use vectors to prove the sine rule.

(b) If θ is the angle between two unit vectors $\underline{a}$ and $\underline{b}$ show that $\frac{1}{2}|\underline{a}+\underline{b}| = \cos\left(\frac{\theta}{2}\right)$.

(c) (i) If $G(t) = e^t\underline{i} + \cos t\,\underline{j} + t\underline{k}$, find $\frac{d}{dt}[(\sin t)G(t)]$.

(ii) Integrate the vector $e^t\underline{i} + 2t\,\underline{j} + \ln t\,\underline{k}$ with respect to t.

(d) Two vectors $\underline{a}$ and $\underline{b}$ have the same magnitude and an angle between them is 60°. If their scalar product is $\frac{1}{2}$, find their magnitude. **(15 marks)**

4. (a) (i) Solve the equation $\log_3 x - 3 + \log_x 9 = 0$.

(ii) The equations $x^2+9x+2=0$ and $x^2+kx+5=0$ have common root. Find the quadratic equation giving two actual possible values of k.

(b) Find the sum of the series $\frac{5}{1\times2\times3}+\frac{8}{2\times3\times4}+\frac{11}{3\times4\times5}+...+\frac{3n+2}{n(n+1)(n+2)}$,

hence find $\sum_{r=1}^{\infty}\frac{3r+2}{r(r+1)(r+2)}$.

(c) If $A=\begin{pmatrix}2 & 1 & 0\\ 1 & 5 & 2\\ 1 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2 & 0\\ 1 & 3 & 2\\ 2 & 0 & 1\end{pmatrix}$, find the value of $A^{-1}B$. **(15 marks)**

SECTION B (40 Marks)

Answer any **two (2)** questions from this section. Extra questions will **not** be marked.

5. (a) (i) Use trigonometric identities to prove that $16\sin^5\theta - 21\sin^3\theta + 5\sin\theta = \sin 5\theta$.

(ii) If $x\sec\theta + y\tan\theta = 3$ and $x\tan\theta + y\sec\theta = 2$, eliminate θ from the equations.

(b) (i) If $\tan\theta = \frac{4}{3}$ and $0° \le \theta \le 360°$, find without using tables the value of $\tan\left(\frac{1}{2}\theta\right)$.

(ii) Show that $\frac{\cos 3x - \cos 5x}{4\sin 2x\cos 2x} = \sin x$.

(c) Given that $\tan^{-1}A + \tan^{-1}B + \tan^{-1}C = \pi$, verify that $A + B + C = ABC$.

(d) Express the sum of $\sec x$ and $\tan x$ as the tangent of $\left(\frac{\pi}{4}+\frac{x}{2}\right)$ and hence find in surd form the value of $\tan\frac{\pi}{12}$.

(20 marks)

6. (a) Define the following terms and write one example for each term:
   (i) Continuous random variable.
   (ii) Discrete random variable
   (iii) Probability density function.

(b) (i) A group of students consist of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has at least a boy and a girl?

(ii) If $P(A)=\frac{1}{4}$, $P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$. Verify whether A *and* B are independent events or are mutually exclusive events.

(c) Rehema and Seni play a game in which Rehema should win 8 games for every 7 games won by Seni. Prove that if they play three games, the probability that Rehema wins at least two games is approximately to 0.55.

(d) In a family, the boy tells a lie in 30 percent cases and the girl in 35 percent cases. Find the probability that both contradict each other on the same fact.

(20 marks)

7. (a) (i) Solve the differential equation $\frac{r\tan\theta}{a^2-r^2}\frac{dr}{d\theta}=1$ given that $r=0$ when $\theta=\frac{\pi}{4}$.

(ii) Verify that $y=10\sin 3x+9\cos 3x$ is a solution of the differential equation $\frac{d^2y}{dx^2}+9y=0$ if $y=0, \frac{dy}{dx}=0$ when $x=0$.

(b) The population of a certain country doubles in 15 years. In how many years will it be six times under the assumption that the rate of increase is proportional to the number of inhabitants?

(c) Find the particular solution of the differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\cos x$.

(d) Form a differential equation whose general solution is $y=Ae^{mx}+Be^{-mx}$ where A, B and m are constants.

(20 marks)

8. (a) (i) The ellipse has its foci at the points $(-1, 0)$ and $(7, 0)$ when its ecentricity is $\frac{1}{2}$. Find its Cartesian equation.

(ii) Convert $y^2=4a(a-x)$ into polar equation.

(iii) Use the equation $y=2x^2-6x+4$ to determine its directrix and the focus.

(b) A cable used to support a swinging bridge approximates the shape of a parabola. Determine the equation of a parabola if the length of the bridge is 100m and the vertical distance from where the cable is attached to the bridge to the lowest point of the cable is 20m.

(c) (i) Define the term hyperbola.

(ii) Show that the latus rectum of the equation $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ is $\frac{2b^2}{a}$.

(d) Sketch the graph of $r=2+4\cos t$.

(20 marks)