THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Time: 3 Hours **Wednesday, 09th May 2018 p.m.**

Instructions

1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) Use Demoivre's theorem to prove that $\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$.

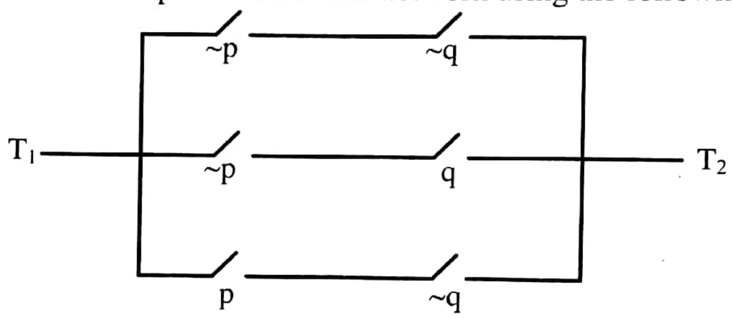
(b) (i) The equation $6 - z^2 = 8i - (2+4i)z$ has roots z_1 and z_2. If $z_1 = 3+i$; find the other root z_2 in form $a+ib$.

(ii) Express $\frac{6}{x^2 - 2x + 10}$ in partial fractions with complex linear denominators.

(c) If z is any complex number, such that $z = r(\cos n\theta + i\sin n\theta)$, use mathematical induction to prove that $z^n = r^n(\cos n\theta + i\sin n\theta)$ for all positive integers n.

(d) If $|z+1| = 2|z-1|$, prove that z lies on a circle whose radius is $\frac{4}{3}$.

2. (a) (i) Draw a simplified electrical network using the following circuit.



(ii) Simplify the proposition $\sim((p \wedge q) \rightarrow (p \vee q))$ by using laws of algebra.

(b) Use the truth table to determine whether $[(p \rightarrow \sim q) \wedge (q \vee r) \wedge p] \rightarrow r$ is tautology.

(c) Use the laws of algebra to prove that the propositions $p \wedge (q \vee r)$ and $[p \rightarrow (q \vee \sim r)] \rightarrow (p \wedge q)$ are equivalent.

3. (a) (i) Find the work done in moving an object along a straight line from $(3,2,-1)$ to $(2,-1,4)$ in a force field given by $F = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$.

(ii) If $\underset{\sim}{a} = (3t+1)\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ is perpendicular to $\underset{\sim}{b} = (t+3)\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$, find the possible values of the constant t.

(b) The vertices of a quadrilateral are A (5, 2, 0), B (2, 6, 1), C (2, 4, 7) and D (5, 0, 6).

(i) Show that the quadrilateral is a parallelogram.

(ii) Find the actual area of the parallelogram in (b) (i).

(c) The vertices A, B and C of the triangle are at the points with position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively. Show that the area of the triangle is equal to $\frac{1}{2}|\underset{\sim}{a} \times \underset{\sim}{b} + \underset{\sim}{b} \times \underset{\sim}{c} + \underset{\sim}{c} \times \underset{\sim}{a}|$ square units.

4. (a) Express $\frac{3x+1}{(x+1)(x^2+2x+3)}$ in partial fractions.

(b) (i) If $a^x = \left(\frac{a}{k}\right)^y = k^m$ where $a \neq 1$; show that $y = \frac{mx}{m-x}$.

(ii) If $x^y = y^{2x}$ and $y^2 = x^3$, solve for x and y.

(c) Expand $\sqrt{1+x}$ as far as the term in x^3 and use the result to obtain the value of $\sqrt{16.08}$ correct to six decimal places.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) (i) If $2\cos\theta = \frac{1}{x} + x$, prove that $2\cos 3\theta = x^3 + \frac{1}{x^3}$.

(ii) Use t-formula to solve the equation $5\cos\alpha - 2\sin\alpha = 2$ for $-180^\circ \leq \alpha \leq 180^\circ$.

(b) (i) If $\theta = \frac{\pi}{8}$ and $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ show that $\tan\left(\frac{1}{8}\pi\right) = \sqrt{2} - 1$.

(ii) Given $a\sin\theta + b\cos\theta = c$, show that $a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$.

(c) (i) Simplify the expression $\tan^{-1} x + \tan^{-1}\left(\frac{1-x}{1+x}\right)$.

(ii) Find all the values of θ which satisfy the equation $\cos x\theta + \cos(x+2)\theta = \cos\theta$.

(d) If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, show that $\frac{a+b+c}{abc} = 1$.

6. (a) If two independent events are A and B such that $P(A) = 2$ and $P(B) = 0.4$, determine

(i) $P(\text{not A and B})$.

(ii) $P(\text{A or B})$.

(b) Kalihose's family consists of mother, father and their ten children. The family is invited to send a group of 4 representatives to a wedding. Find the number of ways in which the group can be formed, if it must contain;

(i) Both parents.

(ii) One parent only.

(iii) None of the parents.

(c) The density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kx & 0 \leq x \leq 1 \\ k(2-x) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find

(i) The value of the constant k,

(ii) $E(X)$,

(iii) $P\left(\frac{1}{2} \leq x \leq 1\frac{1}{2}\right)$.

(d) If X follows binomial distribution with mean 4 and variance 2, find $P(|x-4| \leq 2)$ and write your answer in four significant figures.

7. (a) Form the differential equation by eliminating arbitrary constants, in the equation $Ax^2 + By^2 = 1$.

(b) Solve $(1+y^2)dx = (\tan^{-1} y - x)dy$.

(c) The rate of decrease of the temperature of a body is proportional to the difference between the temperature of the body and that of the surrounding air. If water at temperature 100°C cools in 20 minutes to 78°C in a room of temperature 25°C, find the temperature of water after 30 minutes correctly to two decimal places.

(d) Find the general equation for the equation $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2\sin x$ given that $y = 1$, $\frac{dy}{dx} = 0$ when $x = 0$.

8. (a) Show that the point $B(5, -5)$ lies on the parabola $y^2 = 5x$ and find the equation of the normal to the parabola at the point B in the form $y = mx + c$.

(b) If $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, find c in terms of a, b, m.

(c) (i) Find the rectangular equation of $r = 12(1+\sin\theta)$.

(ii) Sketch the graph of $r = \sin 2\theta$ for $0 \leq \theta \leq \pi$.