ALGEBRA

An algebraic expression – is a collection of numbers, variables, operators and grouping symbols.Variables - are letters used to represent one or more numbers

Algebraic Operations

Symbols to form Algebraic Expressions

Use symbols to form algebraic expressions

The parts of an expression collected together are called **terms**

Example

- x + 2x – are called like terms because they have the same variables
- 5x +9y – are called unlike terms because they have different variables

An algebraic expression can be evaluated by replacing or substituting the numbers in the variables

Example 1

Evaluate the expressions below, given that x = 2 and y = 3

(a) $5x - y$

(b) $3(x + 2)$

(c) $\frac{1}{3}x + 2y$

Solution

(a) $5x - y = 5(2) - 3 = 10 - 3 = 7$

(b) $3(x + 2) = 3(2 + 2) = 3(4) = 12$

(c) $\frac{1}{3}x + 2y = \frac{1}{3}(2) + 2(3) = \frac{2}{3} + 6 = 6\frac{2}{3}$

Example 2

Evaluate the expressions below, given that m = 1 and n = - 2

(a) $\frac{3}{m+1}$

(b) $\frac{m}{1-n}$

(c) $m-n$

Solution

(a) $\frac{3}{m+1} = \frac{3}{1+1} = \frac{3}{2} = 1\frac{1}{2}$

(b) $\frac{m}{1-n} = \frac{1}{1- -2} = \frac{1}{1+2} = \frac{1}{3}$

(c) $m-n = 1- \ -2 = 1+2 = 3$

An expression can also be made from word problems by using letters and numbers

Example 3

A rectangle is 5 cm long and w cm wide. What is its area?

Solution

Let the area be A.

Then

A = length× widith

A = 5w cm^2

Simplifying Algebraic Expressions

Simplify algebraic expressions

Algebraic expressions can be simplified by addition, subtraction, multiplication and division

Addition and **subtraction** of algebraic expression is done by adding or subtracting the coefficients of the like terms or letters

Coefficient of the letter – is the number multiplying the letter

Multiplication and **division** of algebraic expression is done on the coefficients of both like and unlike terms or letters

Example 4

Simplify the expressions below

(a) $4a + 3b + 2a + b$

(b) $6n + 3m - 2n - 2m$

(c) $\frac{1}{2}s - t + 3s - \frac{1}{4}t$

(d) $5mn - 3nm$

Solution

(a) $4a + 3b + 2a + b$

Collect like terms together

$$4a + 3b + 2a + b = 4a + 2a + 3b + b$$
$$= 6a + 4b$$

(b) $6n + 3m - 2n - 2m$

Collect like terms together

$$6n + 3m - 2n - 2m = 6n - 2n + 3m - 2m$$
$$= 4n + m$$

(c) $\frac{1}{2}s - t + 3s - \frac{1}{4}t$

Collect like terms together

$$\frac{1}{2}s - t + 3s - \frac{1}{4}t = \frac{1}{2}s + 3s - t - \frac{1}{4}t$$
$$= 3\frac{1}{2}s - 1\frac{1}{4}t$$

(d) $5mn - 3nm = 5mn - 3mn = 2mn$

Equations with One Unknown

An equation – is a statement that two expressions are equal

An Equation with One Unknown

Solve an equation with one unknown

An equation can have one variable *(unknown)* on one side or two variables on both sides.

When you shift a number or term from one side of equation to another, its sign changes

- If it is positive, it becomes negative
- If it is negative, it becomes positive

Example 5

Solve the following equations

(a) $x + 3 = 5$

(b) $x - 2\frac{1}{4} = 8$

(c) $3x + 4 = -3$

(d) $5x + 11 = 18$

(e) $5 - 3x = 24$

Solution

(a) $x+3=5$

$$x=5-3$$
$$x=2$$

(b) $x-2\frac{1}{4}=8$

$$x=8+2\frac{1}{4}$$
$$x=8+\frac{9}{4}$$
$$x=\frac{8}{1}+\frac{9}{4}=\frac{32+9}{4}=\frac{41}{4}=10.25$$
$$x=10.25$$

(c) $3x+4=-3$

$$3x=-3-4$$
$$3x=-7$$
$$x=-\frac{7}{3}=-2.33 \quad , \quad x=-2.33$$

(d) $5x+11=18$

$$5x=18-11$$
$$5x=7$$
$$x=\frac{7}{5}=1.4$$
$$x=1.4$$

(e) $5-3x=24$

$$-3x=24-5$$
$$-3x=19$$
$$x=\frac{19}{-3}=-\frac{19}{3}=-6.33$$
$$x=-6.33$$

An Equation from Word Problems

Form and solve an equation from word problems

Some word problems can be solved by using equations as shown in the below examples

Example 6

Naomi is 5 years young than Mariana. The total of their ages 33 years. How old is Mariana?

Solution

Let the age of Mariana be x

$$\text{Naomi} = x - 5$$

Then $$x + (x - 5) = 33$$

$$x + x - 5 = 33$$

$$2x = 33 + 5$$

$$2x = 38$$

$$x = \frac{38}{2} = 19$$

$$x = 19$$

Mariana is 19 years

Equations with Two Unknowns

Simultaneous Equations

Solve simultaneous equations

Simultaneous equations – are groups of equations containing multiple variables

Example 7

Examples of simultaneous equation

(ii) $\begin{cases} x + y = 1 \\ 2x - y = -7 \end{cases}$ (ii) $\begin{cases} 4t - 2s = 13 \\ -5t - s = 5 \end{cases}$ (iii) $\begin{cases} w - 9z = 40 \\ \frac{1}{3}w - \frac{1}{8}z = 34 \end{cases}$

A simultaneous equation can be solved by using two methods:

- Elimination method
- Substitution method

ELIMINATION METHOD

STEPS

- Choose a variable to eliminate*e.g x or y*
- Make sure that the letter to be eliminated has the same coefficient in both equations and if not, multiply the equations with appropriate numbers that will give the letter to be eliminated the same coefficient in both equations

(i) $\begin{cases} x - 3y = 8 \\ 2x + 3y = 4 \end{cases}$ eliminate y because it has the same coeffic

(ii) $\begin{cases} x - 5y = -1 \\ x + y = 2 \end{cases}$ eliminate x because it has the same coeffic

(iii) $\begin{cases} 5x + 4y = 2 \\ 3x + y = 3 \end{cases}$

- To eliminate x, multiply equation (i) by 3 and $(i$) by

$i.e$ $\left| \begin{matrix} 3 & 5x + 4y = 2 \\ 5 & 3x + y = 3 \end{matrix} \right.$

$\rightarrow \begin{cases} 15x + 12y = 6 \\ 15x + 5y = 15 \end{cases}$

- To eliminate y, multiply equation (i) by 1 and (ii) by 4

$i.e$

$$\left|\begin{array}{ll} 1 & 5x + 4y = 2 \\ 4 & 3x + y = 3 \end{array}\right.$$

$$\rightarrow \begin{cases} 5x + 4y = 2 \\ 12x + 4y = 12 \end{cases}$$

- If the signs of the letter to be eliminated are the same, subtract the equations
- If the signs of the letter to be eliminated are different, add the equations

Example 8

Solve the following simultaneous equations by elimination method

(a) $\begin{cases} 3x + y = 9 \\ 5x - y = 7 \end{cases}$

(b) $\begin{cases} 3x - 2y = 13 \\ 3x + 2y = 1 \end{cases}$

(c) $\begin{cases} 5r - g = 14 \\ 4r + 3g = 15 \end{cases}$

(d) $\begin{cases} 7x + 6y = 8 \\ 2x - 3y = 7 \end{cases}$

Solution

a. Eliminate y

$$+\left|\begin{array}{l}3x + y = 9 \ldots\ldots\ldots.(i)\\ 5x - y = 7 \ldots\ldots\ldots(ii)\end{array}\right.$$

$$8x = 16$$
$$x = \frac{16}{8} = 2$$
$$x = 2$$

To find y put x = 2 in either equation (i) or (ii)

From equation (i)

$$3x + y = 9$$
$$3(2) + y = 9$$
$$6 + y = 9$$
$$y = 9 - 6$$
$$y = 3$$

$$\therefore\ \ x = 2\ ,\ \ y = 3$$

(b)Eliminate x

$$-\left|\begin{array}{l}3x - 2y = 13 \ldots\ldots\ldots.(i)\\ 3x + 2y = 1 \ldots\ldots\ldots(ii)\end{array}\right.$$

$$-4y = 12$$
$$y = -12$$
$$y = -\frac{12}{4} = -3$$
$$y = -3$$

In order to find y, put x = 2 in either equation (i) or (ii)

From equation (ii)

$$3x + 2y = 1$$
$$3x + 2(-3) = 1$$
$$3x - 6 = 1$$
$$3x = 1 + 6$$
$$3x = 7$$
$$x = \frac{7}{3} = 2\frac{1}{3}$$
$$x = 2\frac{1}{3}$$

$\therefore \; x = 2\frac{1}{3} \;,\;\; y = -3$

(c) Given

$$\begin{cases} 5r - g = 14 \ldots\ldots\ldots (i) \\ 4r + 3g = 15 \ldots\ldots\ldots (ii) \end{cases}$$

Eliminate g

$$\begin{array}{c|l} 3 & 5r - g = 14 \\ 1 & 4r + 3g = 15 \end{array}$$

$$+\left| \begin{array}{l} 15r - 3g = 42 \\ 4r + 3g = 15 \end{array} \right.$$

$$19\,r = 57$$

$$r = \frac{57}{19} = 3$$

$$r = 3$$

To find g put r = 3 in either equation (i) or (ii)

From equation *(i)*

$$5r - g = 14$$

$$5(3) - g = 14$$

$$15 - g = 14$$

$$-g = 14 - 15$$

$$-g = -1$$

$$g = 1$$

$\therefore\ r = 3\ ,\quad g = 1$

(d) Given

$$\begin{cases} 7x + 6y = 8 \dots\dots\dots (i) \\ 2x - 3y = 7 \dots\dots\dots (ii) \end{cases}$$

Eliminate x

$$\begin{array}{c|l} 2 & 7x + 6y = 8 \\ 7 & 2x - 3y = 7 \end{array}$$

$$- \left| \begin{array}{l} 14x + 12y = 16 \\ 14x - 21y = 49 \end{array} \right.$$

$$33y = -33$$

$$y = -\frac{33}{33} = -1$$

$$y = -1$$

To find x, put y = - 1 in either equation(i) or (ii)

From equation *(ii)*

$$2x - 3y = 7$$

$$2x - 3(-1) = 7$$

$$2x + 3 = 7$$

$$2x = 7 - 3$$

$$2x = 4$$

$$x = \frac{4}{2} = 2$$

$\therefore \; x = 2 \;, \quad y = -1$

BY SUBSTITUTION

STEPS

- Make the subject one letter in one of the two equation given e.g $\begin{cases} x - 3y = 8 \ldots\ldots\ldots (\text{i}) \\ 2x + 3y = 4 \ldots\ldots .. (\text{ii}) \end{cases}$

Make x the subject from equation (i)

$$x = 8 + 3y \ldots \ldots . (\text{iii})$$

- Substitute the letter in the remaining equation and proceed as in case of elimination

$$2x + 3y = 4$$
$$2(8 + 3y) + 3y = 4$$
$$16 + 6y + 3y = 4$$
$$16 + 9y = 4$$
$$9y = 4 - 16$$
$$9y = -12$$
$$y = -\frac{12}{9} = -\frac{4}{3}$$

From (iii)

$$x = 8 + 3y$$
$$x = 8 + 3\left(-\frac{4}{3}\right)$$
$$x = 8 - 4 = 4$$

$\therefore \quad x = 4 \quad \text{or} \quad -\frac{4}{3}$

Example 9

Solve the following simultaneous equations by substitution method

(a) $\begin{cases} 2x + y = 17 \\ x + y = 4 \end{cases}$

Solution

(a) $\begin{cases} 2x + y = 17 \ldots\ldots\ldots (\text{i}) \\ x + y = 4 \ldots\ldots\ldots. (\text{ii}) \end{cases}$

From (ii)

$$x = 4 - y \ldots\ldots\ldots\ldots (\text{iii})$$

Substitute (iii) into (i)

$$2x + y = 17$$
$$2(4 - y) + y = 17$$
$$8 - 2y + y = 17$$
$$8 - y = 17$$
$$-y = 17 - 8$$

$$-y = 9$$
$$y = -9$$

From (iii)

$$x = 4 - y$$
$$x = 4 - \ -9$$
$$x = 4 + 9 = 13$$

$\therefore \ x = 13, \quad y = -9$

Linear Simultaneous Equations from Practical Situations

Solve linear simultaneous equations from practical situations

Simultaneous equations can be used to solve problems in real life involving two variables

Example 10

If 3 Mathematics books and 4 English books weighs 24 kg and 5 Mathematics books and 2 English books weighs 20 kg, find the weight of one Mathematics book and one English book.

Solution

Let the weight of one Mathematics book = x and

Let the weight of one English book = y

Then

$$\begin{cases} 3x + 4y = 24 \ldots\ldots\ldots (i) \\ 5x + 2y = 20 \ldots\ldots\ldots (ii) \end{cases}$$

Eliminate y

$$\begin{array}{r|l} 2 & 3x + 4y = 24 \\ 4 & 5x + 2y = 20 \end{array}$$

$$- \left| \begin{array}{r} 6x + 8y = 48 \\ 20x + 8y = 80 \end{array} \right.$$

$$-14x = -32$$

$$x = \frac{-32}{-14} = 2.29$$

$$x = 2.29$$

To find y, put x = 2.29 in either equation (i) or (ii)

From equation*(i).*

$$3x + 4y = 24$$
$$3(2.29) + 4y = 24$$
$$6.87 + 4y = 14$$
$$4y = 24 - 6.87$$
$$4y = 17.13$$
$$y = \frac{17.13}{4} = 4.28$$

$\therefore \ x = 2.29 \ , \quad y = 4.28$

Inequalities

An inequality – is a mathematical statement containing two expressions which are not equal. One expression may be less or greater than the other.The expressions are connected by the inequality symbols$<,>,\leq$ or$\geq$.Where$<$ = less than,$>$ = greater than,$\leq$ = less or equal and $\geq$ = greater or equal.

Linear Inequalities with One Unknown

Solve linear inequalities in one unknown

An inequality can be solved by collecting like terms on one side.Addition and subtraction of the terms in the inequality does not change the direction of the inequality.Multiplication and division of the sides of the inequality by a positive number does not change the direction of the inequality.But multiplication and division of the sides of the inequality by a negative number changes the direction of the inequality

Example 11

Solve the following inequalities

(a) $x+8>15$ (b) $7-2x<11$ (c) $\frac{1}{5}x-3\geq -2+x$ (d) $3x -$

(e) $\frac{2x+8}{-3}\geq 20$ (f) $\frac{1}{2}x-4\leq 3-\frac{2}{3}x$

Solution

(a) $x+8>15$

$$x>15-8$$

$$x>7$$

(b) $7-2x<11$

$$-2x<11-7$$

$$-2x<4$$

$$x>-2$$

(c) $\frac{1}{5}x-3\geq -2+x$

Collect like terms

$$\frac{1}{5}x-x\geq -2+3$$

$$-\frac{4}{5}x\ \geq\ -1$$

$$x\leq \frac{5}{4}$$

(d) $3x - 1 < 2x - 5$

Collect like terms

$$3x - 1 < 2x - 5$$
$$3x - 2x < -5 + 1$$
$$x < -4$$

(e) $\frac{2x+8}{-3} \geq 20$

Multiply by -3 both sides

$$2x + 8 \leq -60$$

$$2x \leq -60 - 8$$

$$2x \leq -68$$

$$x \leq -34$$

(f) $\frac{1}{2}x - 4 \leq 3 - \frac{2}{3}x$

Collect like terms

$$\frac{1}{2}x - 4 \leq 3 - \frac{2}{3}x$$
$$\frac{1}{2}x + \frac{2}{3}x \leq 3 + 4$$
$$\frac{3x + 4x}{6} \leq 3$$
$$\frac{7}{6}x \leq 3$$
$$x \leq 3\left(\frac{6}{7}\right)$$
$$x \leq \frac{18}{7}$$

Linear Inequalities from Practical Situations

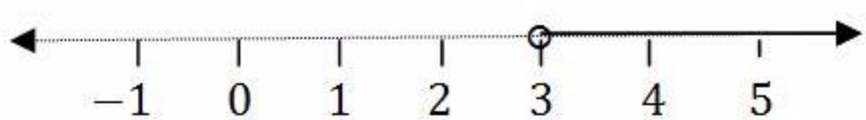
Form linear inequalities from practical situations

To represent an inequality on a number line, the following are important to be considered:

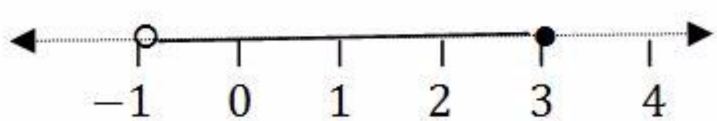
- The endpoint which is not included is marked with an empty circle
- The endpoint which is included is marked with a solid circle

Example 12

(a) $x > 3$



(b) $-1 < x \leq 3$



Compound statement – is a statement made up of two or more inequalities

Example 13

Solve the following compound inequalities and represent the answer on the number line

(a) $10 \le 2x - 3 < 14$

(b) $7 \le 3 - 2x < 15$

Solution

(a) $10 \le 2x - 3 < 14$

Express in the form of $a < x < b$

Add 3 on each part of the inequality

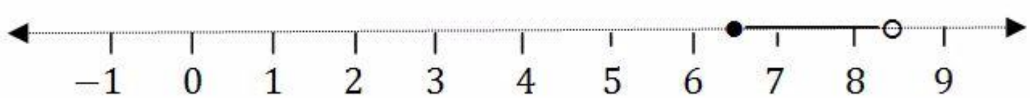
$$10 + 3 \le 2x - 3 + 3 < 14 + 3$$

$$13 \le 2x < 17$$

Divide by 2 each part of the inequality

$$\frac{13}{2} \le x < \frac{17}{2}$$

$$6\frac{1}{2} \le x < 8\frac{1}{2}$$



(b) $7 \leq 3 - 2x < 15$

Subtract 3 on each part of the inequality

$$7 - 3 \leq 3 - 3 - 2x < 15 - 3$$

$$4 \leq -2x < 12$$

Divide by -2 each part of the inequality

$$\frac{4}{-2} \geq x > \frac{12}{-2}$$

$$-2 \geq x > -6$$

$$-6 < x \leq -2$$