

FRACTIONS

A fraction is a number which is expressed in the form of **a/b** where **a** - is the top number called numerator and **b**- is the bottom number called denominator.

Proper, Improper and Mixed Numbers

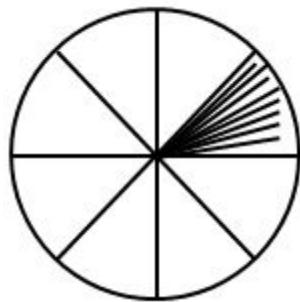
A Fraction

Describe a fraction

A fraction is a number which is expressed in the form of a/b where **a** - is the top number called numerator and **b**- is the bottom number called denominator.

Consider the diagram below

The shaded part in the diagram above is 1 out of 8, hence mathematically it is written as $1/8$



Example 1

(a) 3 out of 5 (three-fifths) = $3/5$

Example 2

(b) 7 Out of 8 (i.e seven-eighths) = $7/8$

Example 3

a. $5/12 = (5 \times 3)/(12 \times 3) = 15/36$

b. $3/8 = (3 \times 2)/(8 \times 2) = 6/16$

Dividing the numerator and denominator by the same number (This method is used to simplify the fraction)

Difference between Proper, Improper Fractions and Mixed Numbers

Distinguish proper, improper fractions and mixed numbers

Proper fraction -is a fraction in which the numerator is less than denominator

Example 4

4/5, 1/2, 11/13

Improper fraction -is a fraction whose numerator is greater than the denominator

Example 5

12/7, 4/3, 65/56

Mixed fraction -is a fraction which consist of a whole number and a proper fraction

Example 6

$1\frac{2}{9}$, $5\frac{35}{203}$, $36\frac{1}{2}$

(a) To convert mixed fractions into improper fractions, use the formula below

(b) To convert improper fractions into mixed fractions, divide the numerator by the denominator

$$\frac{\text{denominator of proper fraction} \times \text{whole number} + \text{numerator of proper fraction}}{\text{denominator of proper fraction}}$$

Example 7

Convert the following mixed numbers into improper fractions

(a) $11\frac{2}{3}$ (b) $6\frac{1}{7}$ (c) $3\frac{7}{8}$ (d) $16\frac{4}{9}$ (e) $6\frac{7}{11}$

Comparison of Fractions

In order to find which fraction is greater than the other, put them over a common denominator, and then the greater fraction is the one with greater numerator.

A Fraction to its Lowest Terms

Simplify a fraction to its lowest terms

Example 8

For the pair of fractions below, find which is greater

(a) $\frac{7}{9}, \frac{6}{7}$ (b) $\frac{3}{8}, \frac{1}{3}$

Solution

(a) $\frac{7}{9} = \frac{7 \times 7}{9 \times 7} = \frac{49}{63}$ and $\frac{6}{7} = \frac{6 \times 9}{7 \times 9} = \frac{54}{63}$

$\therefore \frac{6}{7}$ is greater than $\frac{7}{9}$

(b) $\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$ and $\frac{1}{3} = \frac{1 \times 8}{3 \times 8} = \frac{8}{24}$

$\therefore \frac{3}{8}$ is greater than $\frac{1}{3}$

Equivalent Fractions

Identify equivalent fractions

Equivalent Fraction

- Are equal fractions written with different denominators
- They are obtained by two methods

(a) Multiplying the numerator and denominator by the same number

(i) $\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$

(ii) $\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}$

(a) Dividing the numerator and denominator by the same number (This method is used to simplify the fraction)

(i) $\frac{14^7}{18^9} = \frac{7}{9}$ (both numerator and denominator are divided by 2)

(ii) $\frac{40^5}{88^{11}} = \frac{5}{11}$ (both numerator and denominator are divided by 8)

NOTE: The fraction which cannot be simplified more is said to be in its lowest form

Example 9

Simplify the following fractions to their lowest terms

$$(a) \frac{25}{40} \quad (b) \frac{26}{36} \quad (c) \frac{13}{39} \quad (d) \frac{100}{80} \quad (e) \frac{45}{120}$$

Solution

$$(a) \frac{\cancel{25}^5}{\cancel{40}^8} = \frac{5}{8} \quad (\text{each number divided by 5})$$

$$(b) \frac{\cancel{26}^{13}}{\cancel{36}^{18}} = \frac{13}{18} \quad (\text{each number divided by 2})$$

$$(c) \frac{\cancel{13}^1}{\cancel{39}^3} = \frac{1}{3} \quad (\text{each number divided by 13})$$

$$(d) \frac{\cancel{100}^5}{\cancel{80}^4} = \frac{5}{4} \quad (\text{each number divided by 20})$$

$$(e) \frac{\cancel{45}^3}{\cancel{120}^8} = \frac{3}{8} \quad (\text{each number divided by 15})$$

Fractions in Order of Size

Arrange fractions in order of size

Example 10

Arrange in order of size, starting with the smallest, the fraction

$$\frac{2}{3}, \quad \frac{4}{7}, \quad \frac{3}{8}, \quad \frac{5}{9}$$

Solution

Put them over the same denominator, that is find the L.C.M of 3, 7, 8 and 9

Operations and Fractions

Addition of Fractions

Add fractions

Operations on fractions involves **addition, subtraction, multiplication** and **division**

- Addition and subtraction of fractions is done by putting both fractions under the same denominator and then add or subtract
- Multiplication of fractions is done by multiplying the numerator of the first fraction with the numerator of the second fraction, and the denominator of the first fraction with the denominator of the second fraction.
- For mixed fractions, convert them first into improper fractions and then multiply
- Division of fractions is done by taking the first fraction and then multiply with the reciprocal of the second fraction
- For mixed fractions, convert them first into improper fractions and then divide

Example 11

Find

$$(a) \frac{1}{5} + \frac{1}{4} \quad (b) \frac{5}{7} + \frac{2}{3} \quad (c) \frac{2}{5} + \frac{7}{13}$$

Solution

$$(a) \frac{1}{5} = \frac{1 \times 4}{5 \times 4} = \frac{4}{20} \quad \text{and} \quad \frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

$$\therefore \frac{1}{5} + \frac{1}{4} = \frac{4}{20} + \frac{5}{20} = \frac{9}{20}$$

$$(b) \frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21} \quad \text{and} \quad \frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

$$\therefore \frac{5}{7} + \frac{2}{3} = \frac{15}{21} + \frac{14}{21} = \frac{29}{21} = 1\frac{8}{21}$$

$$(c) \frac{2}{5} = \frac{2 \times 13}{5 \times 13} = \frac{26}{65} \quad \text{and} \quad \frac{7}{13} = \frac{7 \times 5}{13 \times 5} = \frac{35}{65}$$

$$\therefore \frac{2}{5} + \frac{7}{13} = \frac{26}{65} + \frac{35}{65} = \frac{61}{65}$$

Subtraction of Fractions

Subtract fractions

Example 12

Evaluate

$$(a) \frac{2}{3} - \frac{3}{8} \quad (b) \frac{3}{4} - \frac{1}{5} \quad (c) \frac{5}{16} - \frac{1}{8}$$

Solution

$$(a) \frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24} \quad \text{and} \quad \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$$\therefore \frac{2}{3} - \frac{3}{8} = \frac{16}{24} - \frac{9}{24} = \frac{7}{24}$$

$$(b) \frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20} \quad \text{and} \quad \frac{1}{5} = \frac{1 \times 4}{5 \times 4} = \frac{4}{20}$$

$$\therefore \frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20} = \frac{11}{20}$$

$$(c) \frac{5}{16} = \frac{5 \times 8}{16 \times 8} = \frac{40}{128} \quad \text{and} \quad \frac{1}{8} = \frac{1 \times 16}{8 \times 16} = \frac{16}{128}$$

$$\therefore \frac{5}{16} - \frac{1}{8} = \frac{40}{128} - \frac{16}{128} = \frac{24}{128}$$

Multiplication of Fractions

Multiply fractions

Example 13

$$\text{Find (a) } \frac{2}{3} \times \frac{3}{4} \quad (b) \frac{15}{16} \times \frac{24}{25} \quad (c) \frac{3}{11} \times 4$$

Solution

$$(a) \frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

$$(b) \frac{15}{16} \times \frac{24}{25} = \frac{15 \times 24}{16 \times 25} = \frac{360}{400} = \frac{9}{10}$$

$$(c) \frac{3}{11} \times 4 = \frac{3}{11} \times \frac{4}{1} = \frac{3 \times 4}{11 \times 1} = \frac{12}{11} = 1 \frac{1}{11}$$

Division of Fractions

Divide fractions

Example 14

Evaluate (a) $\frac{3}{5} \div \frac{2}{3}$ (b) $\frac{23}{36} \div \frac{45}{48}$ (c) $\frac{5}{7} \div 8$

Solution

$$\begin{aligned} \text{(a)} \quad & \frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{3 \times 3}{5 \times 2} = \frac{9}{10} \\ \text{(b)} \quad & \frac{23}{36} \div \frac{45}{48} = \frac{23}{36} \times \frac{48}{45} = \frac{23 \times 48}{36 \times 45} = \frac{1104}{1520} = \frac{69}{95} \\ \text{(c)} \quad & \frac{5}{7} \div 8 = \frac{5}{7} \div \frac{8}{1} = \frac{5}{7} \times \frac{1}{8} = \frac{5 \times 1}{7 \times 8} = \frac{5}{56} \end{aligned}$$

Mixed Operations on Fractions

Perform mixed operations on fractions

Example 15

Find (a) $2\frac{3}{5} + 3\frac{3}{4}$ (b) $9\frac{1}{2} - 4\frac{5}{8}$

Solution

$$\begin{aligned} \text{(a)} \quad & 2\frac{3}{5} = \frac{13}{5} = \frac{13 \times 4}{5 \times 4} = \frac{52}{20} \quad \text{and} \quad 3\frac{3}{4} = \frac{15}{4} = \frac{15 \times 5}{4 \times 5} = \frac{75}{20} \\ \therefore & 2\frac{3}{5} + 3\frac{3}{4} = \frac{52}{20} + \frac{75}{20} = \frac{127}{20} = 6\frac{7}{20} \\ \text{(b)} \quad & 9\frac{1}{2} = \frac{19}{2} = \frac{19 \times 8}{2 \times 8} = \frac{152}{16} \quad \text{and} \quad 4\frac{5}{8} = \frac{37}{8} = \frac{37 \times 2}{8 \times 2} = \frac{74}{16} \\ \therefore & 9\frac{1}{2} - 4\frac{5}{8} = \frac{152}{16} - \frac{74}{16} = \frac{78}{16} = 4\frac{14}{16} \end{aligned}$$

Example 16

Evaluate (a) $5\frac{2}{3} \div 1\frac{1}{2}$ (b) $4\frac{3}{5} \times 7\frac{1}{8}$

Solution

$$(a) 5\frac{2}{3} \div 1\frac{1}{2} = \frac{17}{3} \div \frac{3}{2} = \frac{17}{3} \times \frac{2}{3} = \frac{17 \times 2}{3 \times 3} = \frac{34}{9} = 3\frac{7}{9}$$

$$(b) 4\frac{3}{5} \times 7\frac{1}{8} = \frac{23}{5} \times \frac{57}{8} = \frac{23 \times 57}{5 \times 8} = \frac{1311}{40} = 32\frac{31}{40}$$

Word Problems Involving Fractions

Solve word problems involving fractions

Example 17

1. Musa is years old. His father is $3\frac{3}{4}$ times as old as he is. How old is his father?
2. $1\frac{3}{4}$ of a material are needed to make suit. How many suits can be made from